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Patterns of Stress Corrosion: Geometry of the Principal Stresses

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Abstract - We investigate migration of fluids through the rock in the stress corrosion process. The fluids migrate along the trajectories of the principal stress field. We study geometry of these trajectories, including singularities, limit cycles, and possible bifurcations. We describe corresponding configurations of weakened zones in the lithosphere due to the fluid migration.

Key words: Stress corrosion; Principal stress; Singularities.

1. *Introduction*

Fluids in the lithosphere may influence the mechanical properties of rocks—such as strength, effective viscosity, etc.—and consequently may cause changes in the properties of the earth before the occurrence of large earthquakes. Several ways in which these influences can take place are described in ANDERSON and GREW (1977), BARENBLATT *et al.* (1981), GELFAND *et al.* (1976), O'NEIL and HANKS (1980), PERTSOV and KOGAN (1981), PERTSOV *et al.* (1977), and RICE (1979). In this paper we study one of these mechanisms, stress corrosion. The existence and importance of stress corrosion in natural rocks is discussed in PERTSOV and KOGAN (1981) and in PERTSOV *et al.* (1977).

The main properties of stress corrosion, as relevant to the present study, may be summarized as follows:

Stress corrosion occurs in a variety of rocks that are permeated by fluids, under conditions such as exist widely in the lithosphere. Fluids migrate through the rock in a direction perpendicular to that of maximum strain or minimum compression. In the course of migration, fractures are opened and filled with fluid; this weakens the rocks and reduces the effective viscosity.

If a rock consists of firm grains imbedded in a relatively softer matrix, then the fractures migrate along grain boundaries; propagation usually takes

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place along grain boundaries whose orientation is close to perpendicular to the axis of maximum strain.

Additional detailed data on stress corrosion in rocks and similar materials may be found in ANDERSON and GREW (1977), PERTSOV and KOGAN (1981), and RICE (1979). Whether there is stress corrosion in the lithosphere, with its high hydrostatic pressure, is a question that is still unanswered. There are several qualitative arguments, supported in part by theoretical considerations (BARENBLATT and CHRISTIANOVICH, 1968), that stress corrosion under conditions of high compression, if it exists, arises as a consequence of a different mechanism than stress corrosion that is produced under conditions of tension. Some field, laboratory, and theoretical studies, however (HALLBAUER *et al.*, 1973; KACHANOV, 1982; NEMAT-NASSER and HORII, 1982; SIBSON, 1981), describe weakening under compression that exhibits the same characteristics as tension-caused stress corrosion. Specifically, even under hydrostatic compression, the early stages of weakening are dominated by tensile cracks oriented perpendicular to the direction of maximum tension. This property has also been observed for dry fracture with no stress corrosion (HALLBAUER *et al.*, 1973; KACHANOV, 1982; NEMAT-NASSER and HORII, 1982). Thus the relations between fractures that are oriented in directions related to the geometry of the principal stresses is a subject worth further investigation. We assume here that stress corrosion in the lithosphere, or some other equivalent mechanism, will lead to crack migrations oriented as above. Under this assumption, we analyze the inhomogeneities of the strength of solids that are due to such migrations.

If the stresses are inhomogeneous, then the geometry of the principal stress and the corresponding trajectories of migrating fluids become rather complicated—singularities, limit cycles, etc. appear. We study the general properties of the principal stresses and the trajectories of migration. We confine our analysis to two-dimensional problems, which may correspond to the migration of fluids in the rupture zones separating two lithospheric blocks. We use methods similar to those of the qualitative theory of dynamical systems (ANDRONOV *et al.*, 1966; 1967). There are, however, fundamental differences between the singularities of the principal stress fields and those of dynamical systems.

Generally the principal stress field may have singularities of the three types that appear in differential geometry, mentioned by geometers of the last century (CAYLEY, 1863; FROST, 1870), and studied recently by PORTEOUS (1971) and THORNDIKE, *et al.* (1981). Combinations of these singularities and limit cycles determine typical patterns of the trajectories of migrating fluids and the corresponding inhomogeneities of the weakening of the medium, which we call quasistatic fatigue. We also study the possible bifurcations of these configurations for time-dependent fields. The results obtained provide a possible, but not unique, explanation of some observed regularities in earthquake occurrence.

2. *The two-dimensional case: Singular points*

Let $x = (x_1, x_2)$ be a coordinate system in R^2 and let $\sigma(x)$ be the stress field, which is a symmetric 2×2 matrix depending smoothly on x. We are interested in the field of the directions of the eigenvectors of $\sigma(x)$, which we can call the "lines of force" of the eigenvectors in analogy with electrostatics. This quantity does not change if we add a matrix $g(x)\mathrm{E}$ to $\sigma(x)$, where E is the unit matrix; therefore we may assume that tr $\sigma(x) = 0$ and

$$\sigma(x) = \begin{pmatrix} \alpha(x) & \beta(x) \\ \beta(x) & -\alpha(x) \end{pmatrix},$$

where $\alpha(x)$ and $\beta(x)$ are smooth functions. The eigenvectors of $\sigma(x)$ are indefinite, and their directional field is singular at the points where two eigenvalues of $\sigma(x)$ coincide, i.e., where $\alpha(x) = \beta(x) = 0$. We call these points *singular*. For a general $\sigma(x)$ the quantity

$$\Delta(x) = \frac{\partial\alpha}{\partial x_1}\frac{\partial\beta}{\partial x_2} - \frac{\partial\alpha}{\partial x_2}\frac{\partial\beta}{\partial x_1}$$

is not zero at singular points. If σ changes with time, $\sigma = \sigma(x, t)$ then singular points with $\Delta = 0$ may appear at some values of t. It is easy to show that near these times two singular points with opposite signs of Δ collide at a point where $\Delta = 0$ and then mutually annihilate; vice versa, two singular points are born from a point with $\Delta = 0$ that appears at a nonsingular place.

As an example of the above consider the stress field

$$\sigma(x, t) = \begin{pmatrix} x_2^2 - t & x_1 \\ x_1 & t - x_2^2 \end{pmatrix}.$$

For $t < 0$ there are no singular points. For $t > 0$ two singular points $(0, \pm\sqrt{t})$ are created.

For a more detailed study of singular points we need formulas for the eigenvalues and eigenvectors of σ. The eigenvalues of σ are

$$\lambda^{\pm} = \pm\{[\alpha(x)]^2 + [\beta(x)]^2\}^{1/2}. \tag{1}$$

The corresponding eigenvectors $e^{\pm}(x)$ are defined by equations

$$e_1^{\pm}(x) \cdot (\alpha(x) - \lambda^{\pm}(x)) + e_2^{\pm}(x) \cdot \beta(x) = 0 \tag{2}$$

or

$$e_1^{\pm}(x) \cdot \beta(x) - e_2^{\pm}(x) \cdot (\alpha(x) + \lambda^{\pm}(x)) = 0. \tag{2'}$$

If x goes around a singular point x_0, then the eigenvectors make a half turn round the origin in the same direction if $\Delta(x_0) > 0$ and in the opposite direction if $\Delta(x_0) < 0$; this follows from (2) and (2′).

To study the lines of force of $e^{\pm}(x)$ near a singular point x_0, we have to find the trajectories of this field entering the point x_0. The tangent vectors $\tau^{\pm}$ to these trajectories at x_0 are defined by the relation

$$e^{\pm}(x_0 + \epsilon\tau^{\pm}) \to \tau^{\pm} \quad \text{for } \epsilon \to 0, \quad \epsilon > 0.$$

It is easily seen that if τ^+ is the solution for the $e^+(x)$ field, then $\tau^- = -\tau^+$ is the solution for the $e^-(x)$ field. From (1) and (2) we have a homogeneous cubic equation for $\tau^{\pm} = (\tau_1, \tau_2)$:

$$(\tau_1{}^2 - \tau_2{}^2) \cdot (\text{grad}\, \beta(x_0), \tau) = 2\tau_1\tau_2 \cdot (\text{grad}\, \alpha(x_0), \tau). \tag{3}$$

For general $\sigma(x)$ this equation has no multiple roots and its discriminant

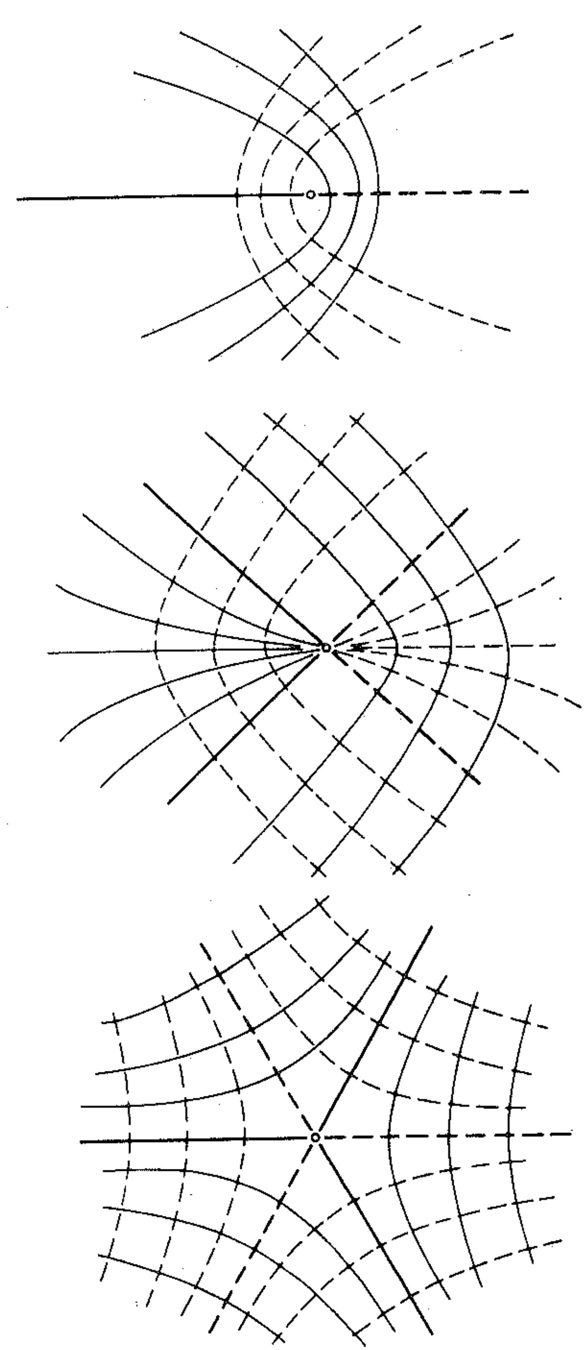
$$\delta(x) = \left\{ 9 \frac{\partial\beta}{\partial x_1}\frac{\partial\beta}{\partial x_2} - \left(\frac{\partial\beta}{\partial x_1} + 2\frac{\partial\alpha}{\partial x_2}\right)\left(\frac{\partial\beta}{\partial x_2} - 2\frac{\partial\alpha}{\partial x_1}\right)\right\}^2 - 4\left\{\frac{\partial\beta}{\partial x_1}\left(\frac{\partial\beta}{\partial x_1} + 2\frac{\partial\alpha}{\partial x_2}\right) + \left(\frac{\partial\beta}{\partial x_2} - 2\frac{\partial\alpha}{\partial x_1}\right)^2\right\} \cdot \left\{\frac{\partial\beta}{\partial x_2}\left(\frac{\partial\beta}{\partial x_2} - 2\frac{\partial\alpha}{\partial x_1}\right) + \left(\frac{\partial\beta}{\partial x_1} + 2\frac{\partial\alpha_1}{\partial x_2}\right)^2\right\}$$

is nonzero at x_0. Equation (3) has a unique solution up to multiplication by any nonzero number for $\delta(x_0) > 0$ and three solutions for $\delta(x_0) < 0$. More detailed considerations show that for $\Delta(x_0) < 0$ we always have $\delta(x_0) < 0$, and neither the corresponding three vectors τ^+ nor the complementary vectors τ^- lie in half planes. The family of curves of lines of force—i.e., the field of directions of $e^{\pm}(x)$ near x_0—is shown in Figure 1a. For $\Delta(x_0) > 0$ there are two cases, $\delta(x_0) < 0$ and $\delta(x_0) > 0$ (see Fig. 1b, c). For $\delta(x_0) < 0$ three vectors τ^+ belong to a certain half plane and three vectors τ^- to the opposite half plane. Any pair of τ^+, as well as τ^-, forms an acute angle for $\Delta(x_0) > 0$ and an obtuse angle for $\Delta(x_0) < 0$. There are special trajectories, called *separatrices*, that start from the singular point x_0, and separate families of trajectories with different behavior near x_0. In cases (a), (b), and (c) there are three, two, and one separatrix, respectively.

3. *The two-dimensional case: Bifurcations of singular points*

For $\sigma = \sigma(x, t)$ evolving in time there are two possible types of bifurcations.

(A) At $t = t_0$ there is one singular point x_0 for $\Delta(x_0) = 0$. A pair of singular points with opposite signs of Δ is either created or annihilated (see Example 1). For $\Delta < 0$ we always have $\delta < 0$. Consequently, at a point with $\Delta > 0$ we also have $\delta < 0$. This means that the singular points are of either type (a) or type (b) (see Fig. 1). There are two possible patterns of bifurcation of the family of trajectories of the field $e^{\pm}(x)$. Figure 2 shows the pattern corresponding to Example 1. A second pattern is obtained by interchanging the families of the trajectories of $e^+(x)$ and $e^-(x)$.



c b a

Figure 1. Lines of force of the eigenvectors $e^+(x)$ (solid lines) and $e^-(x)$ (dashed lines) near a singular point x_0. The separatrices are indicated by heavy lines. (a) $\Delta(x_0) < 0$, (b) $\Delta(x_0) > 0$, $\delta(x_0) < 0$; (c) $\Delta(x_0) > 0$, $\delta(x_0) > 0$.

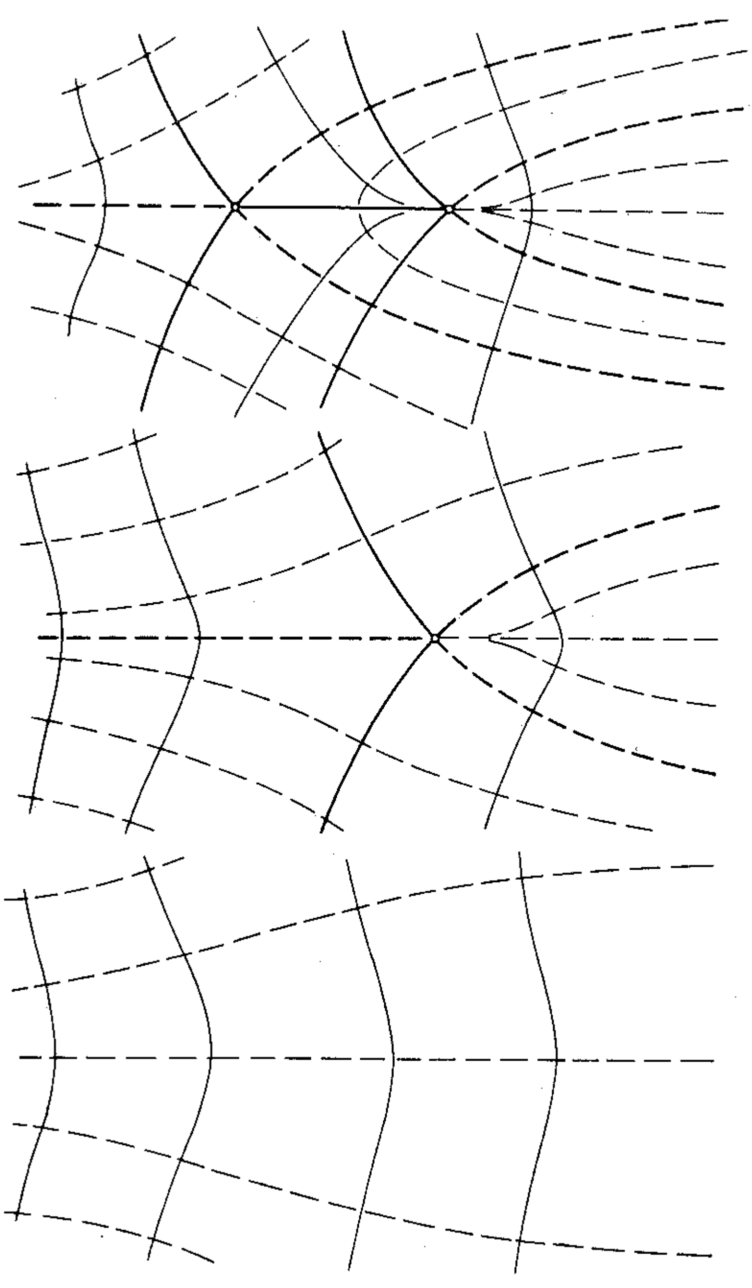


Figure 2. Bifurcation of the family of lines of force of eigenvectors $e^+(x)$ (solid lines) and $e^-(x)$ (dashed lines) with *the birth of a pair of* singular points, (a) $t < t_0$, (b) $t = t_0$, and (c) $t > t_0$.

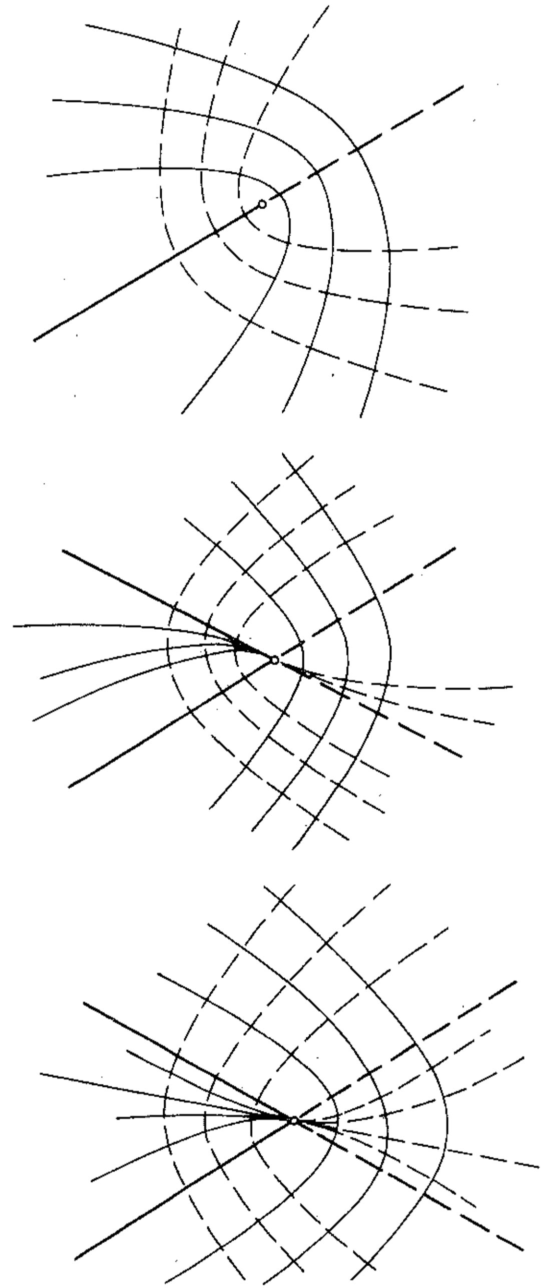


Figure 3. Bifurcation of the family of lines of force of eigenvectors $e^+(x)$ (solid lines) and $e^-(x)$ (dashed lines) with change of sign of $\delta(x_0)$, (a) $t < t_0$, $\delta(x_0) < 0$; (b) $t = t_0$, $\delta(x_0) = 0$; (c) $t > t_0$, $\delta(x_0) > 0$.

(B) At $t = t_0$ a singular point x_0 appears with $\delta(x_0) = 0$. Then $\Delta(x_0) > 0$ and for values of t close to t_0 a singular point of type (b) changes into a singular point of type (c), or vice versa. The corresponding transformation of the family of trajectories of the lines of force of $e^{\pm}(x)$ is shown in Figure 3.

4. The two-dimensional case: Nonlocal topology

The topology of the lines of force of the eigenvectors $e^{\pm}(x)$ can also be characterized by closed trajectories, or limit cycles. Such a cycle necessarily contains singular points (see Fig. 4). If we circumnavigate a limit cycle in some direction, then nearby trajectories wind on or off the cycle. There are relationships between singular points and limit cycles that give us other topological characteristics of the lines of force. In particular, some limit cycles may circumscribe other limit cycles and singular points. Then a trajectory that starts from a singular point or winds off a limit cycle may travel to another singular point, or wind on another limit cycle, or may go off to infinity. Generally, the topology of the lines of force of eigenvectors is described in a manner that is completely analogous to that of dynamical systems (see ANDRONOV *et al.*, 1966); only the singular points differ in the two cases. The characteristic nonlocal bifurcations of the lines of force of eigenvectors evolving in time are also analogous to those of dynamical

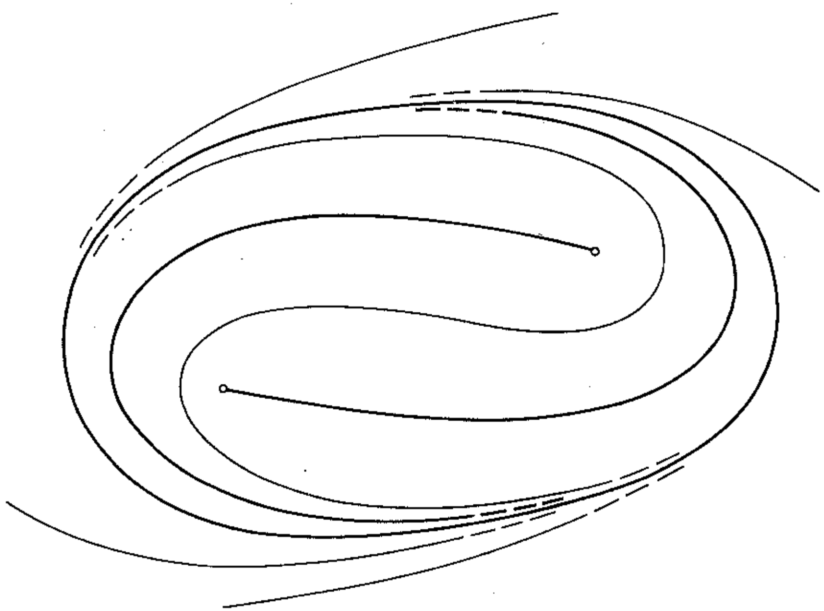


Figure 4. Limit cycle with a pair of singular points in the interior.

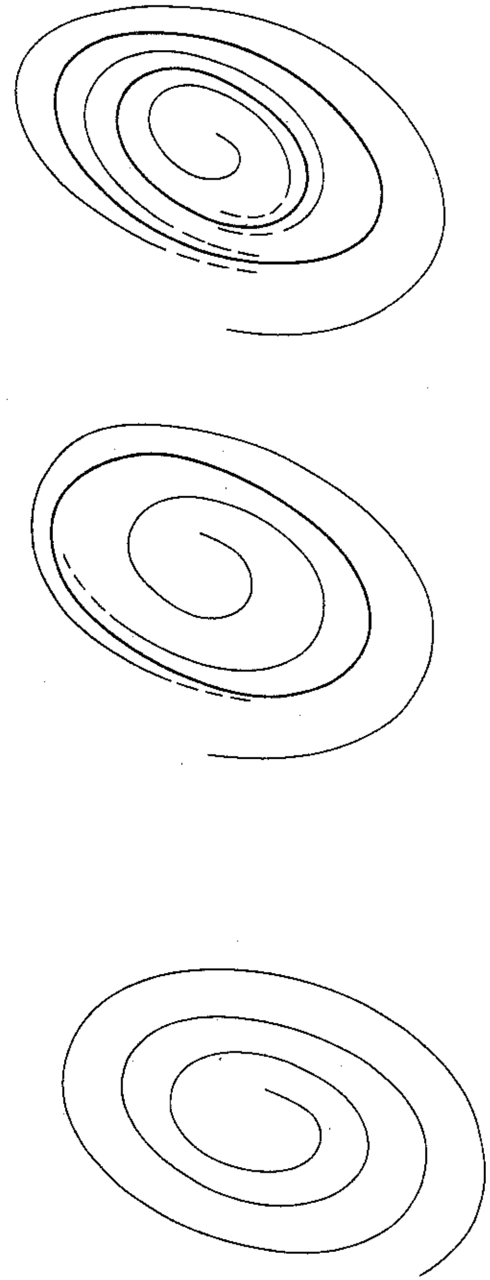


Figure 5. Birth of a pair of limit cycles. (a) $t < t_0$, (b) $t = t_0$, (c) $t > t_0$.

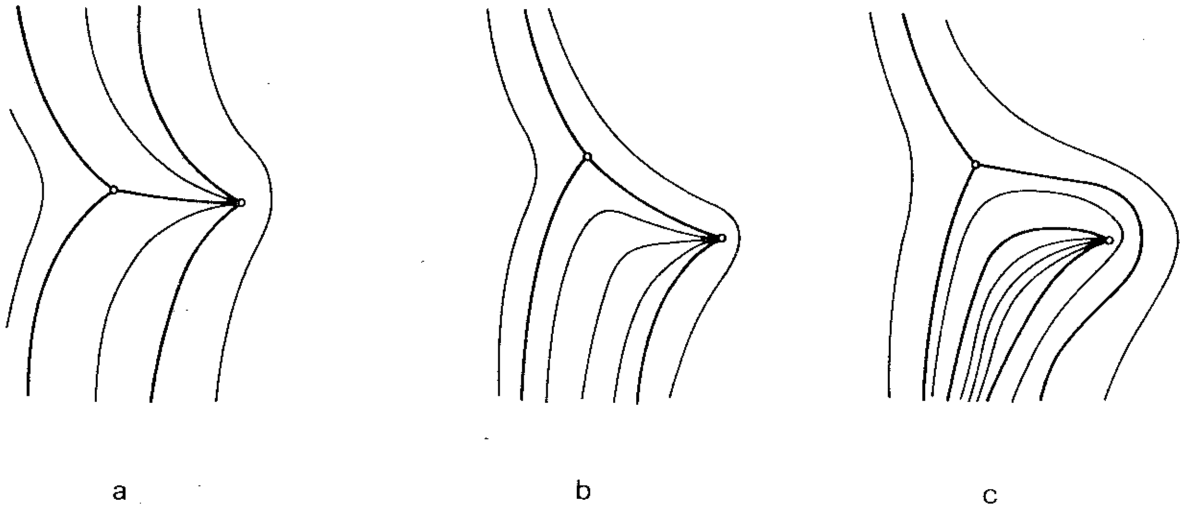


Figure 6. Bifurcation of a separatrix. (a) $t < t_0$, (b) $t = t_0$, (c) $t > t_0$.

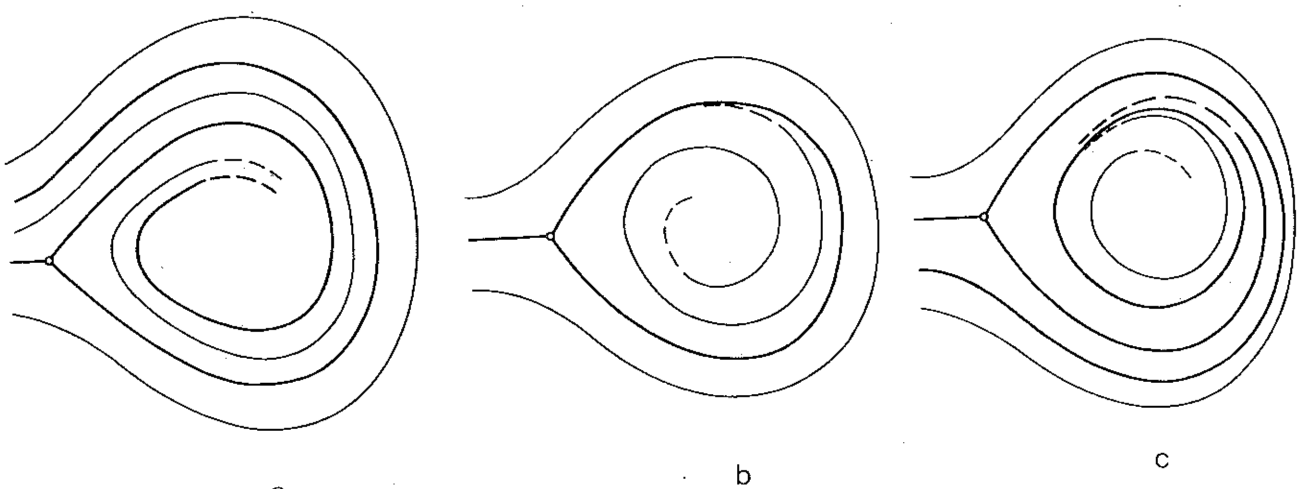


Figure 7. Bifurcation of a separatrix, accompanied by the birth of a limit cycle born out of a loop of the separatrix. (a) $t < t_0$, (b) $t = t_0$, (c) $t > t_0$.

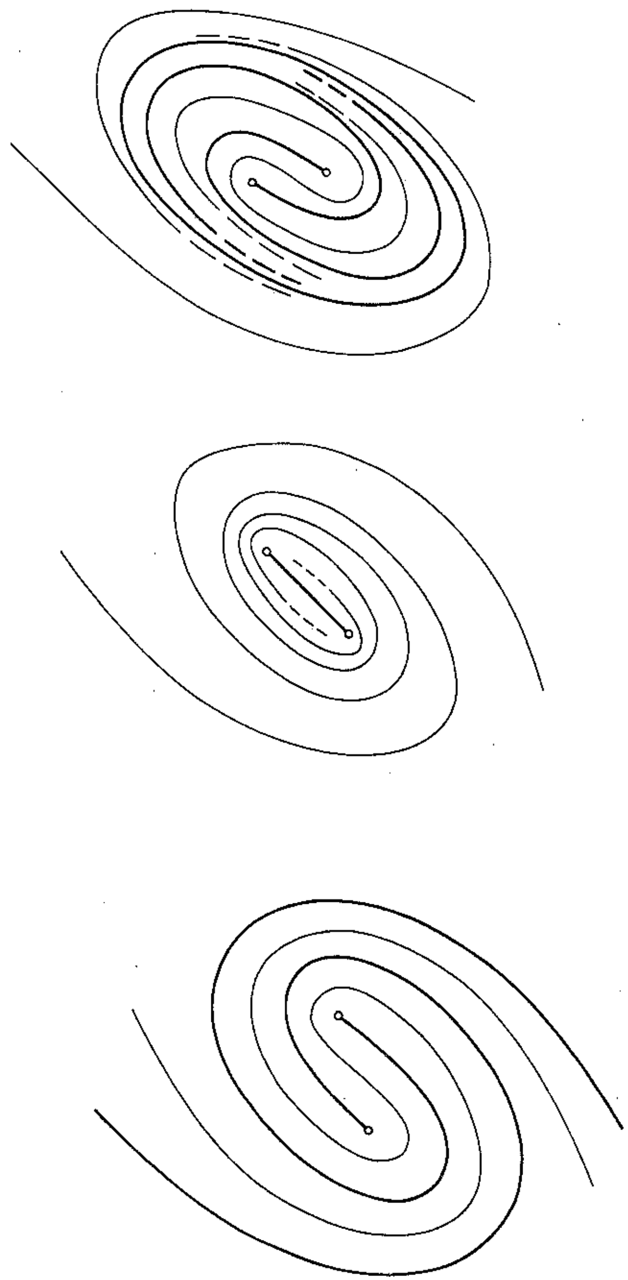


Figure 8. Bifurcation of a separatrix; birth of a limit cycle from a segment of the separatrix.

systems (see ANDRONOV *et al.*, 1967). The evolution of these topological systems may have the following features:

A. A singular point may go off to infinity or may emerge from infinity.
B. Two nearby limit cycles with the opposite direction of winding of trajectories may be born or may mutually annihilate (See Fig. 5).
C. Separatrix bifurcation occurs when a separatrix starting from a singular point arrives at another (or at the same) singular point. In this case, a limit cycle may appear or be destroyed (see examples in Figs. 6, 7, and 8).

5. *The three-dimensional case*

The space of symmetric 3×3 matrices is six-dimensional and matrices with two coinciding eigenvalues form a subvariety of codimension two with singularities on the subvariety of codimension five formed by the matrices with three equal eigenvalues. Thus, for general $\sigma(x)$ depending on x, its singular set (i.e., the set of coincidences of two of its eigenvalues) is a nonsingular curve in R^3. For σ evolving in time, $\sigma = \sigma(x, t)$ its singular set is a nonsingular two-dimensional surface $S \subset R^4_{x,t}$, and at some moment bifurcations of the curves $S_{t_0} = S \cap \{t = t_0\}$ may appear. These bifurcations correspond to critical points of the function t on the surface S. These may be of three types (see Fig. 9):

(a) If t has a minimum, a little closed loop is born.
(b) If t has a saddle point, we have a hyperbolic bifurcation,
(c) If t has a maximum, a little closed loop disappears.

Let $\lambda_1(x) \geq \lambda_2(x) \geq \lambda_3(x)$ be the eigenvalues of the matrix $\sigma(x)$, and $e_1(x)$, $e_2(x)$, $e_3(x)$ the corresponding eigenvectors. Suppose $\lambda_1(x) = \lambda_2(x)$ on a singular curve. To study the singularities of the field of planes orthogonal to $e_1(x)$ in the vicinity of a singular curve we consider the intersection of these planes with planes drawn at points of the singular curve perpendicular to $e_3(x)$. We obtain two-dimensional fields with singularities as described above. In particular, we have segments of the curve with the same (a), (b), and (c) types of singularities as in Figure 1. At the ends of the segments there are bifurcations from (a) to (b)—$e_3(x)$ being orthogonal to the singular curve—and from (b) to (c). However, this does not completely describe the topology of the field of planes near a singular curve. The problem of the complete description has not as yet been solved.

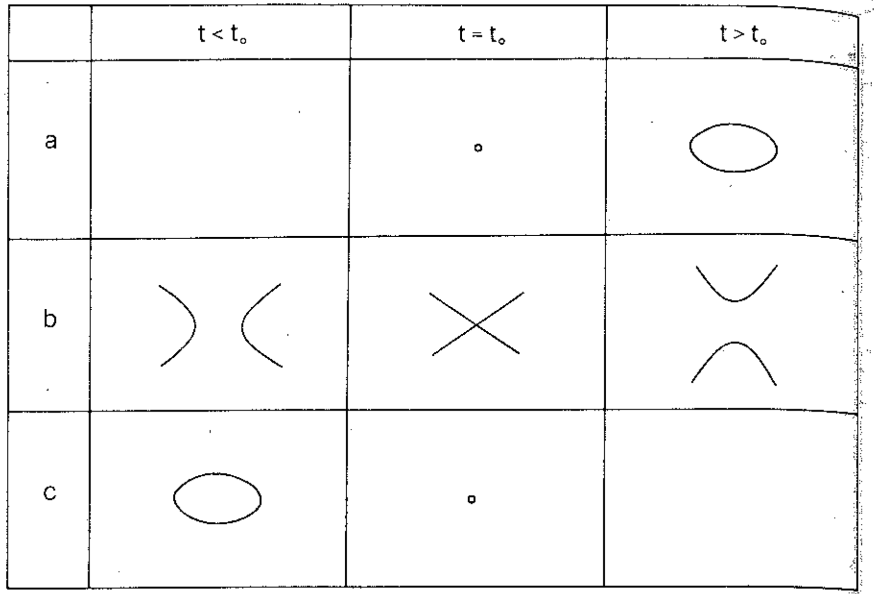


Figure 9. Bifurcations of the singular set in the three-dimensional case: (a) birth of a loop; (b) bifurcation of hyperbolas; (c) annihilation of a loop.

6. *The matrix $\sigma(x)$ for elastic media*

The matrix $\sigma(x)$ is not arbitrary. There are restrictions on it posed by the equations of elasticity:

$$\sum_j \frac{\partial \sigma_{ij}}{\partial x_j} + f_j = 0,$$

where $f(x)$ is a body force density. These restrictions are not, however, significant for our study of the singularities, and for the following reason: The direction field of the eigenvectors of $\sigma(x)$ does not change if we add a matrix $g(x) \cdot \mathbf{E}$ to $\sigma(\mathbf{E})$, $\mathbf{E}$ being the unit matrix. This means that all of the conditions on $\sigma(x)$ that appear in our problem depend only on the nondiagonal elements of $\sigma(x)$, and on pairwise differences between its diagonal elements, as well as on the derivatives of these nondiagonal elements and differences between diagonal elements. This implies that any subvariety in the matrix jet space defined by some condition on the singularity of the field of planes orthogonal to $e_1(x)$ is transverse to the subvariety defined by the elastic equations. According to THOM's transversality theorem (1955–56, 1969) this means that the general singularities and their bifurcations of matrices satisfying the elasticity equations are also general for arbitrary

matrices. It also can be shown that all the singularities and their bifurcations considered above may be obtained for matrices $\sigma(x)$ satisfying the elasticity equations. For example, by adding $(x_2^2 - x_2) \cdot \mathbf{E}$ to the matrix of the example in section 2, we obtain a matrix satisfying the homogeneous equations of elasticity and thus have the same bifurcation structure as in Figure 2.

7. *Geometry of weakened zones in the lithosphere*

Let us assume that a source of fluid is in contact with a part of a boundary of a two-dimensional region $\boldsymbol{\Omega}$. Further, suppose that a stress distribution $\sigma(x)$ in Ω is given. Migration of the fluid according to the mechanism of stress

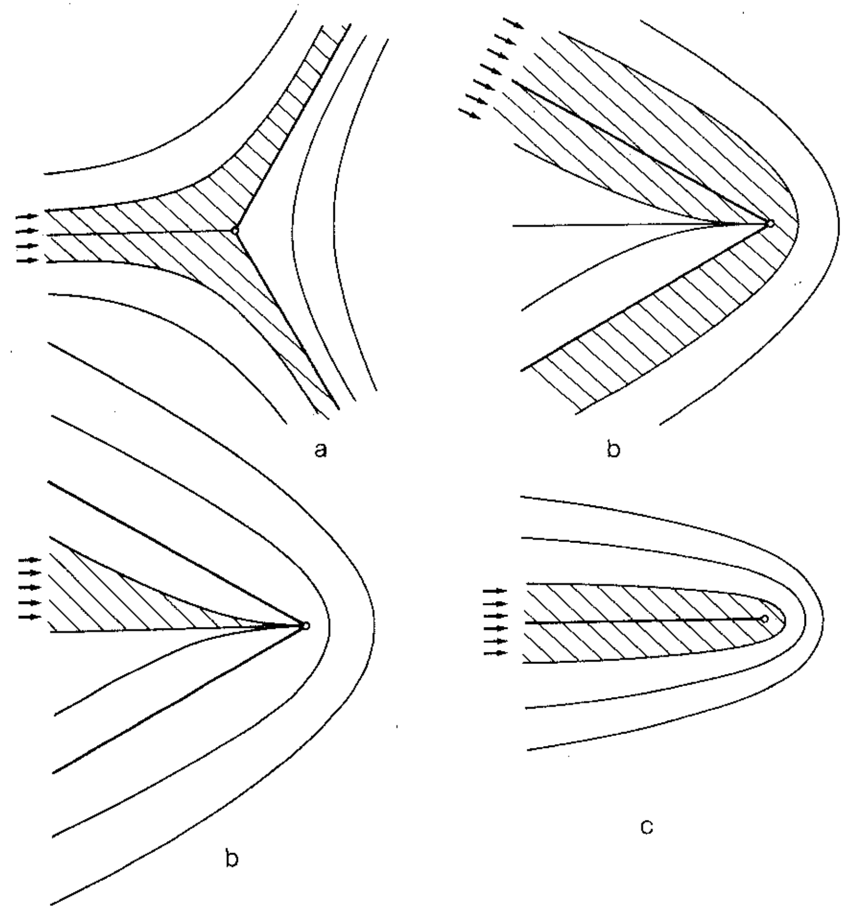


Figure 10. Weakened zone (hatched) near singular points, (a), (b), and (c) being singular points of the types shown in Figure 1. The source of the fluid is marked by arrows.

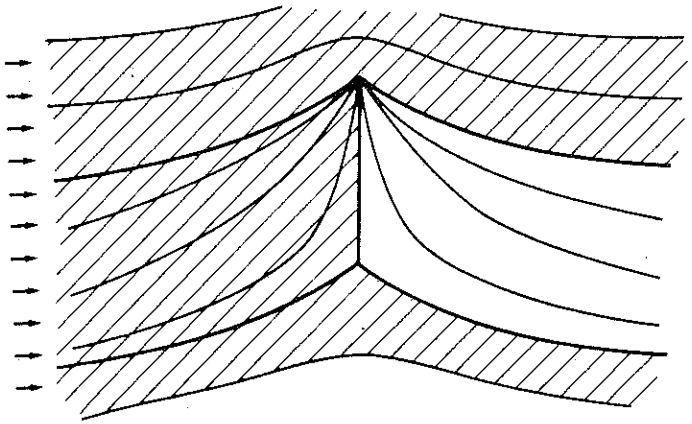


Figure 11. Weakened zone (hatched) near a combination of singular points. The source of the fluid is marked by arrows.

corrosion causes microfractures, and therefore weakening, of some region $\Omega_1 \subset \Omega$. The geometry of that region depends essentially on the properties of the field of principal stresses considered above. It seems natural to assume that the weakened zone is formed by trajectories of migration that originate at the source—i.e., by trajectories of the direction field of maximum compression. Such a weakened zone will have a complicated configuration near singular points and limit cycles of the field. The weakened zone near singular points is shown in Figure 10. Figure 11 presents the weakened zone for the combination of singular points that corresponds to Figure 2c. The weakened zone near a limit cycle is shown in Figure 12.

Some remarks should be made on the possible correspondence of these configurations to reality. First, stress corrosion takes place, according to PERTSOV and KOGAN (1981) and RHEBINDER and SHTCHUKIN (1972), only if the deviatoric tensile stress exceeds some threshold. Therefore the trajectories of fluid migration will not enter a small neighborhood of a singular point, since the deviatoric stress is zero at such a point, by definition. The weakened zone shown in Figures 10 to 12 will therefore contract. Then in the medium there is a slow diffusion transverse to the trajectories, and our configurations therefore broaden somewhat. In particular, the alternation of weakened and nonweakened layers in the vicinity of a limit cycle (Fig. 12) may not take place. We remind the reader that the influence of the weakening on the stress field has not been taken into account here.

Finally, while the results of this study are not specific to stress corrosion but can be applied to any model of fractures that migrate along the axes of maximum compression, the existence of such fractures under high hydrostatic pressure remains a subject of controversy. In two-dimensional cases

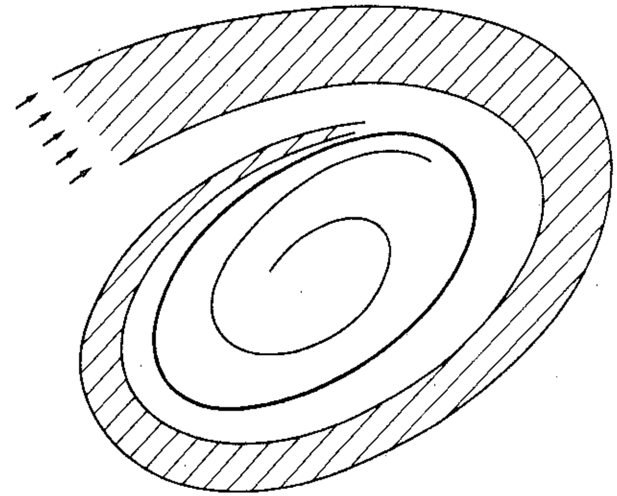


Figure 12. Weakened zone (hatched) near a limit cycle. The source of the fluid is marked by arrows.

our results apply also to a mechanism of stress corrosion, wherein fractures migrate along the axes of maximum shear stress.

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Crack Fusion as a Model for Repetitive Seismicity

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Abstract - A renormalization group treatment of a skeletal, hierarchical model of crack fusion and suturing yields the experimental stress versus time-to-fracture law. We construct a model that eliminates the intervening size states in the hierarchy but retains the time delay that represents the time-to-fracture observations. We add a source term to replenish microcracks lost by promotion due to fusion. The system is found to be stable for all values of time delay if the rate of replenishment is steady. If we allow the rate of replenishment of microcracks to be coupled to the rate of appearance of the largest size cracks, which we interpret as large-scale seismicity, then a Hopf bifurcation appears and the system is describable as a limit cycle attractor.

Key words: Limit cycle; Attractor; Renormalization group; Seismicity; Creep; Bifurcation.

1. Introduction

The time span of most good earthquake catalogs is woefully short when compared with the recurrence times of large earthquakes. For example, in a region such as Southern California, a "good" catalog that lists most events with magnitudes $M \geq 4$ spans the most recent 50 years. If the recurrence rate of earthquakes with $M \geq 8$ is on the order of 100 years or much longer, some assumptions must be made that would permit extrapolation from the time series of the past 50 years to possibly long times in the future. The assumptions most commonly made for this purpose are: (a) that the magnitude-frequency relation, which is an average across 50 years of events for Southern California and across other time intervals for other regions, can be extended to very large magnitudes; and (b) that the earthquakes occur as Poisson independent events. The mere existence of aftershocks indicates that *some* earthquakes are not Poisson independent events. Additional recent seismological information casts doubt on the validity of the second of these assumptions. We describe an attempt to incorporate some of the available seismological and laboratory information in the development of a model

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