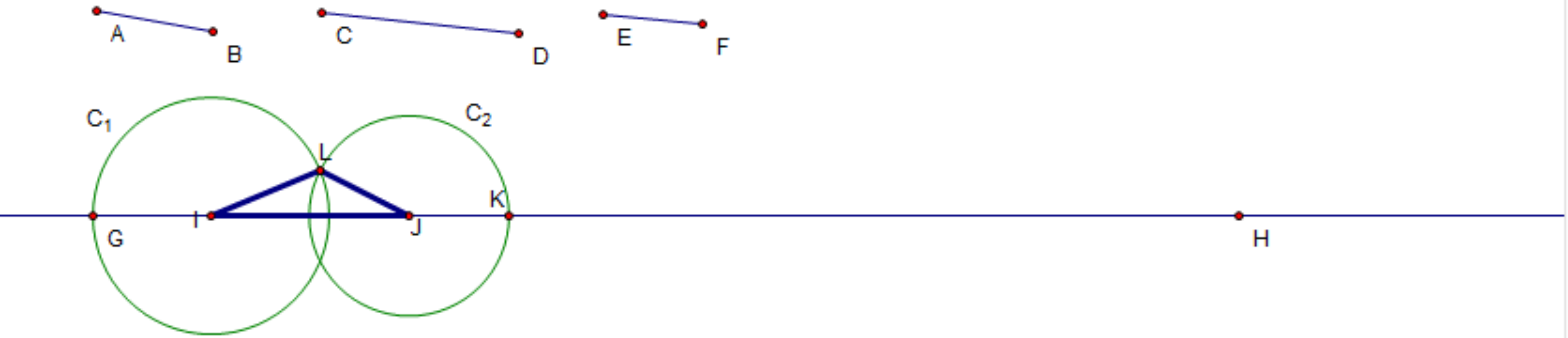
Proposition 22: *Given 3 line segments with the sum of any two greater than the third, we can construct a triangle whose sides are equal in length to the three given sides.*

Proof: Let AB, CD, and EF be three line segments with $AB+CD > EF$, $AB+EF > CD$, and $CD+EF > AB$. We need to construct a triangle whose sides are lengths AB, CD, and EF, respectively.

Let $\overleftrightarrow{GH}$ be any line, and on $\overrightarrow{GH}$ pick points I, J, K with $GI=AB$, $IJ=CD$, and $JK=EF$. We can do this by Proposition 3. Using Postulate 2, draw the circle C_1 with center I and radius IG, and the circle C_2 with center J and radius JK. Let L be a point on the intersection of C_1 and C_2. Draw IL and JL, by Postulate 1. Then, $\triangle IJL$ has $IL=IG=AB$, $IJ=CD$, and $JL=JK=EF$ *(all by CN1)*. Thus, we have constructed $\triangle IJL$ whose sides have the three given lengths. QEF



NOTE: Euclid proves more than he claims in the statement. In fact the statement of the proposition should be: *Given 3 line segments with the sum of any two greater than the third, we can construct a triangle whose sides are equal in length to the three given sides, with any two of the three sides having endpoints at a given point on a given line.*

In fact this is how Euclid applies the result, as we'll see.