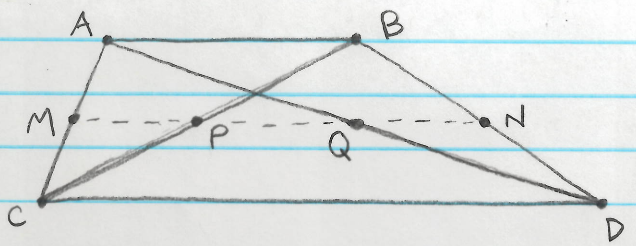
Grant Doyle HW 10, #13

Given: ABCD is a trapezoid with AB ∥ DC and
M, N, P, Q are midpoints of the obvious segments.
Prove that M, N, P, and Q all lie on the same line.



- Consider △CBA. M is midpoint of CA, P is midpoint of CB. By Thm 18, MP ∥ AB.
- MP ∥ AB and AB ∥ CD → MP ∥ CD by BF 14.
- Consider △ACD, M is midpoint of CA, Q is midpoint of AD. By Thm 18, MQ ∥ CD.
- MP ∥ CD and MQ ∥ CD → $\overleftrightarrow{MP}$ ~~∥~~ = $\overleftrightarrow{MQ}$ by BF ~~14~~ 13.
- So M, P, and Q are on the same line.
- Consider △DBA. N is midpoint of BD and Q is midpoint of AD. By Thm 18, QN ∥ AB.
- QN ∥ AB and CD ∥ AB → QN ∥ CD by BF 14.
- Consider △BCD. P is midpoint of BC and N is midpoint of BD. By Thm 18, PN ∥ CD.
- QN ∥ CD and PN ∥ CD → QN ∥ PN by BF ~~14~~ 13.
- So N, P, and Q must be on same line.
- Since M, P, and Q are on same line and N, P, and Q are on same line, then M, N, P, and Q all lie on the same line as claimed. BF7

Q.E.D.