5. (See Figure 4) Let C be a circle with center O and suppose A is on C. Let P be any point other than O. Prove that the circle K with **diameter** OP contains the midpoint M of any segment AB such that B is on C and the line AB contains P. **Hint:** Use what you know about M to show that ΔPOM is a right triangle. Then use what you know about the center of the circumcenter of a right triangle.

HW12 #5

Deng Pan

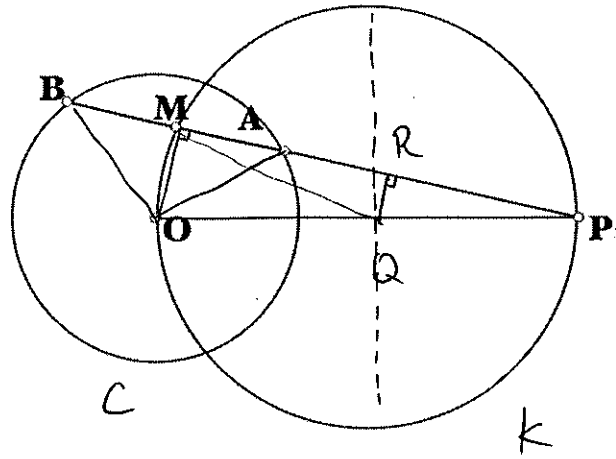

Figure 4

prove: connect AO, BO, MO. Given M is midpoint of AB

By def of midpoint, $MA = MB$. We have $OM = OM$.

By def of circle, $AO = BO$. By BF1, $\triangle AOM \cong \triangle BOM$.

By def of congruence, $\angle OMB = \angle OMA$.

By theorem 1a and algebra, $\angle OMB = \angle OMA = 90°$, right angle.

now, draw point Q be midpoint of OP, draw $QR \perp MP$ at R

By theorem 4, $\angle P = \angle P$, $\angle QRP = \angle OMP \Rightarrow \angle RQP = \angle MOP$

By def of similar $\triangle$, $\triangle PQR \sim \triangle POM$

By BF4, $PQ = \frac{1}{2} PO \Rightarrow PR = \frac{1}{2} PM$

so that RQ is perpendicular bisector of MP.

since Q is midpoint of OP, we are able to draw perpendicular bisector of OP through Q

We can see that perpendicular bisector of OP intersects RQ. Q

By theorem 24, Q is the intersection of three perpendicular bisector of $\triangle$ POM

Therefore Q is the circumcenter of $\triangle$ OMP

By theorem 25, $QO = QP = QM$

By def of circle, Q is the center of circle $\triangle$ OMP.

Since $QO = QP$, and OQP collinear, OP is the diameter of circle K. QED

Note: OR we can prove $QO = QP = QM$ By connecting QM and prove $\triangle QMR \cong \triangle QPR$, and use def of circumcenter.