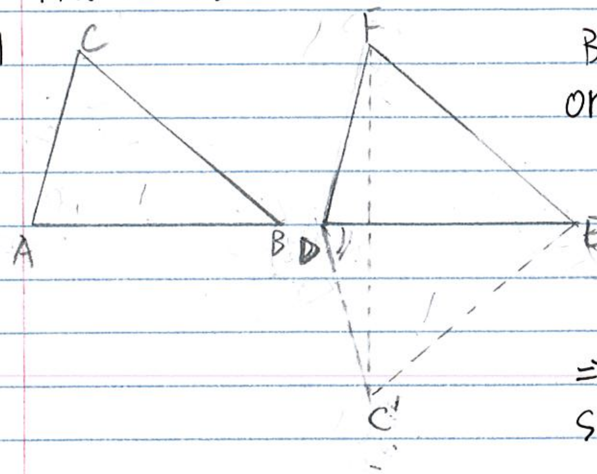
Zijin Liu.

Prove Prop. 8 without using proof by contradiction, or idea of "applying".

HW 12 #8

Case 1



By Prop 23. we can construct an angle on DE, which equals to $\angle CAB$. (below DE)

By Prop 3. we can cut off $DC' = AC$

Then connect ~~DC'~~, FC', EC'.

$C'D = AC$, $\angle C'DE = \angle CAB$, $DE = AB$

$\Rightarrow \triangle C'DE \cong \triangle CAB$ by Prop 4.

So, $C'E = CB$, Then $C'E = EF$, $C'D = DF$ by Common notion 1.

By Prop 5, $\angle DFC' = \angle DC'F$, $\angle EFC' = \angle EC'F$

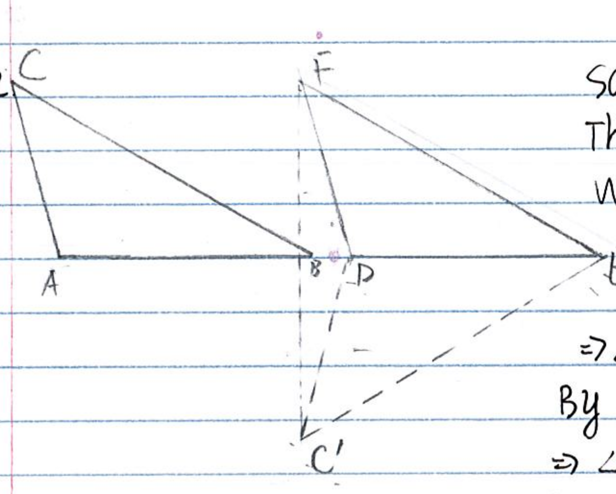
By common notion 2, $\angle DFC' + \angle EFC' = \angle DC'F + \angle EC'F$

$\Rightarrow \angle EC'D = \angle EFD$.

By Common notion 1, $\angle EC'D = \angle BCA = \angle EFD$.

Now by Prop 4, $\triangle ACB \cong \triangle DFE$.

Case 2



Same as Case 1, we construct $C'D$.

Then connect $C'D$, $C'E$.

We can prove that $\triangle C'DE \cong \triangle CAB$ by a similar argument.

Then $C'D = DF$, $C'E = EF$ by common notion 1.

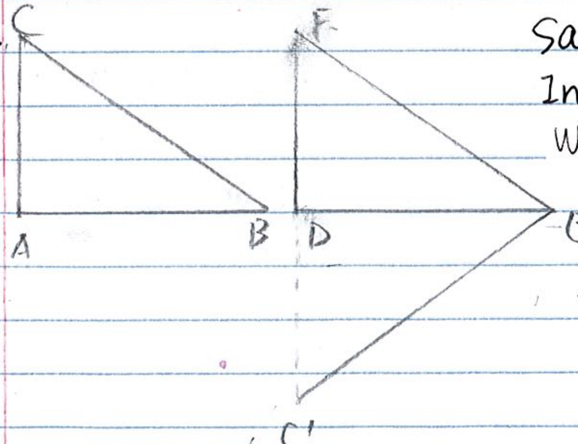
$\Rightarrow \angle DFC' = \angle DC'F$, $\angle EFC' = \angle EC'F$ by Prop 5.

By common notion 2, $\angle EFC' - \angle DFC' = \angle EC'F - \angle DC'F$

$\Rightarrow \angle EC'D = \angle EFD = \angle BCA$ by common notion 1

By Prop 4. $\triangle ACB \cong \triangle DFE$.

Case 3.



Same as Case 1, construct $C'D$, then connect $C'E$.

In this case, C', D, F are collinear.

We can prove that $EF = EC'$ by congruence.

By Prop 5, $\angle F = \angle C'$

By common notion 1, $\angle C' = \angle C = \angle F$.

Then $\triangle ACB \cong \triangle DFE$ by Prop 4. Q.E.D