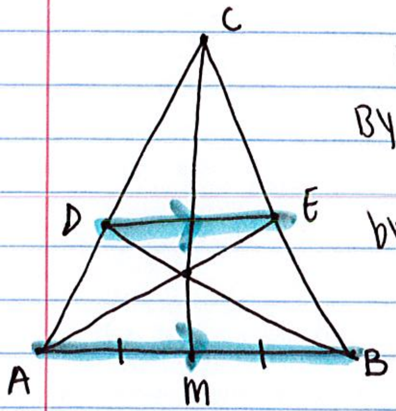
Isabelle Muller

HW 12 #9

Given: M is the midpoint of AB, and the lines that look concurrent are concurrent.

To prove: DE is parallel to AB.



Proof: By the given, BD, AE, CM are concurrent.

By thm 36, $\left[\frac{DA}{DC}\right]\left[\frac{EC}{EB}\right]\left[\frac{MB}{MA}\right] = -1$

by def of midpoint, $\left[\frac{MB}{MA}\right] = -1$

$$\left(\frac{DA}{DC}\right)\left(\frac{EC}{EB}\right) = 1$$

by algebra, $\left(\frac{DA}{DC}\right) = \left(\frac{EB}{EC}\right)$ (*)

by BF6 and algebra,

$$\left(\frac{AC}{DC}\right) = \left(\frac{AD+DC}{DC}\right) = \left(\frac{DC}{DC}\right) + \left(\frac{AD}{DC}\right) = 1 + \left(\frac{AD}{DC}\right) = 1 + \left(\frac{EB}{EC}\right) \quad \text{by } (*)$$

$$= \left(\frac{EC}{EC}\right) + \left(\frac{EB}{EC}\right) = \left(\frac{EC+EB}{EC}\right) = \left(\frac{BC}{EC}\right)$$

$\frac{AC}{DC} = \frac{BC}{EC}$ and $\angle ACB = \angle DCE$

by thm 19, $\triangle ABC \sim \triangle DEC$

by def of similar triangles, $\angle CDE = \angle CAB$

by BF5, DE || AB.

QED