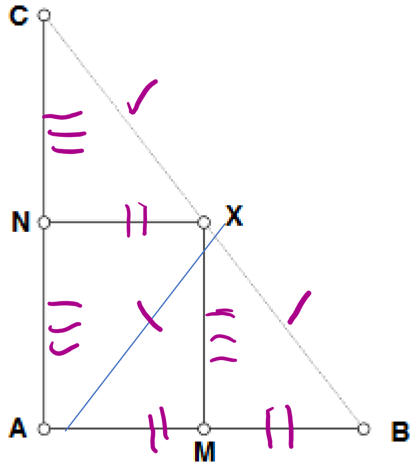
Answer 3



Draw AX.

To prove: $\angle CXA = \angle BXA = 90°$ (i.e, $AX \perp BC$) and $CX = BX$.

Proof:

In ΔAMX and ΔBMX,

$AM = BM$ (as M is midpoint of AB)

$MX = MX$ (common)

$\angle AMX = \angle BMX = 90°$ (as $XM \perp AB$)

Hence, by BF1, $\Delta AMX \cong \Delta BMX$, and $AX = BX$ by definition of congruent triangles.

Similarly, taking ΔCNX and ΔANX, we get $\Delta CNX \cong \Delta ANX$ and $CX = AX$.

But as $AX = BX$ and $AX = CX$, by algebra, $AX = BX = CX$.

But as $CX = BX$, by definition of midpoint, X is the midpoint of CB.

Note (DG): The proof is essentially done here!! (WHY)

Now, in ΔCAB, N is the midpoint of CA and X is the midpoint of BC. Hence, by Theorem 18, $NX \parallel AB$ and $NX = \frac{1}{2}AB$.

But as M is the midpoint of AB, by Theorem 16, AB = 2 AM. But as NX = ½ AB, by algebra, AB = 2 NX and hence AM = NX.

By a similar argument on ΔBAC, we get $XM = \frac{1}{2}AC$ and hence $XM = AN$.

Now, taking ΔXNC and ΔBMX,

$NC = MX$ (as $XM = AN$ and $CN = AN$ and hence by algebra)

$XN = BM$ (as $NX = AM$ and $AM = MB$ and hence by algebra)

$CX = XB$ (as proved earlier)

Hence, by BF1, $\Delta CNX \cong \Delta BMX$, and $\angle CXN = \angle MXB$ by definition of congruent triangles.

And taking ΔANX and ΔXMA, as $AN = XM$, $NX = MA$ and $XA = AX$, by BF1, $\Delta ANX \cong \Delta XMA$. By definition of congruent triangles, $\angle NXA = \angle AXM$.

But as $\angle CXA = \angle CXN + \angle NXA$ and $\angle AXB = \angle BXM + \angle AXM$, as $\angle CXN = \angle MXB$ and $\angle NXA = \angle AXM$, by algebra, it follows that $\angle CXA = \angle BXA$.

But by BF6, $\angle CXA + \angle BXA = 180°$, and hence by algebra, it follows that $\angle CXA = \angle BXA = 90°$, as required.