

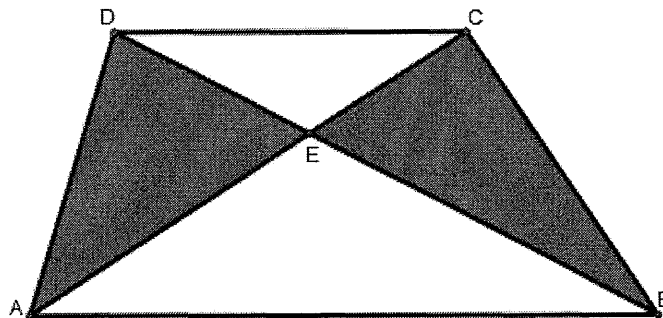
HW 7 Problem 7

We are given  $CD \parallel AB$ .

We want to show that  $\text{Area}(\triangle AED) = \text{Area}(\triangle BEC)$ .

By Theorem 22,  $\text{Area}(\triangle AED) = \frac{1}{2} AE DE \sin(\angle AED)$

By Theorem 22,  $\text{Area}(\triangle BEC) = \frac{1}{2} BE CE \sin(\angle BEC)$



As  $AC$  crosses  $BD$  at  $E$ , by theorem 1 b),  $\angle AED = \angle BEC$ .

Now we want to show  $AE DE = BE CE$ .

As  $DC \parallel AB$ , with  $AC$  and  $BD$  intersecting transversals,

by Theorem C,  $\triangle AEB \sim \triangle CED$ .

Then by Basic Fact 4,  $\frac{BE}{DE} = \frac{AE}{CE}$ .

By algebra,  $AE DE = BE CE$ .

Then by transitivity,

$$\text{Area}(\triangle AED) = \frac{1}{2} AE DE \sin(\angle AED) = \frac{1}{2} BE CE \sin(\angle BEC) = \text{Area}(\triangle BEC)$$

$\text{Area}(\triangle AED) = \text{Area}(\triangle BEC)$ .

