Homework 8: Problem 8

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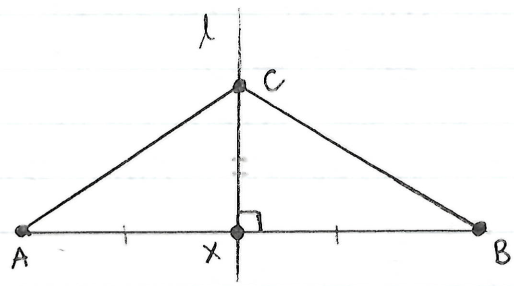
a) Given a segment AB and a point C (which may or may not be on AB).

Prove: C is on the perpendicular bisector of AB $\Leftrightarrow$ AC = BC.

Note: (DG)

You must consider the case when C lies on AB separately in (i) and (ii)



i) C is on the perpendicular bisector of AB

NTS AC = BC

l is perpendicular bisector (C is on l) of AB, X is intersection

Draw AC and BC.

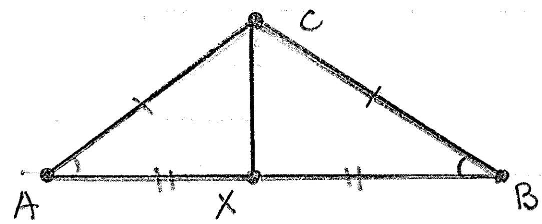
Definition of perpendicular bisector, AX = BX

Definition of perpendicular line, $\angle AXC = 90° = \angle BXC$

CX = CX

BF 2, $\triangle AXC \cong \triangle BXC$

Definition of congruence, AC = BC



ii) $AC = BC$

NTS C is on the perpendicular bisector of AB

Theorem 5a, $\measuredangle A = \measuredangle B$

BF 10, let X be the midpoint of AB.

Definition of midpoint, $AX = BX$

Draw CX

BF 2, $\triangle AXC \cong \triangle BXC$

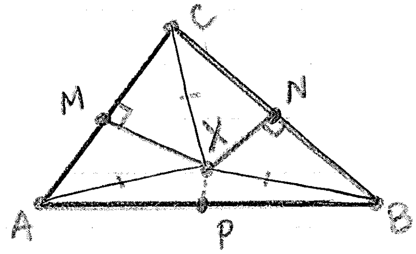
Definition of congruence, $\measuredangle AXC = \measuredangle BXC$

Theorem 1a and algebra, $\measuredangle AXC = 90° = \measuredangle BXC$

CX is the perpendicular bisector of AB by definition, and C is on CX

b) Use part a) to give a shorter proof of Theorem 24. Begin as usual by constructing two perpendicular bisectors and their intersection X. Then connect X to the three verticies. After that you may not use any Basic Fact or Theorem.

Theorem 24: For any triangle, the perpendicular bisectors are concurrent.



Proof:

Let ABC be a triangle

BF 10, construct midpoints of sides

BF 12, draw perpendicular lines through two of the midpoints and let X be the intersection

MX and NX are the perpendicular bisectors by definition

From a) $AX = CX$ and $BX = CX$, so by algebra $AX = BX$

From a) X is on the perpendicular bisector of AB

X is a shared point among the three perpendicular bisectors of $\triangle ABC$, therefore they are concurrent ∎