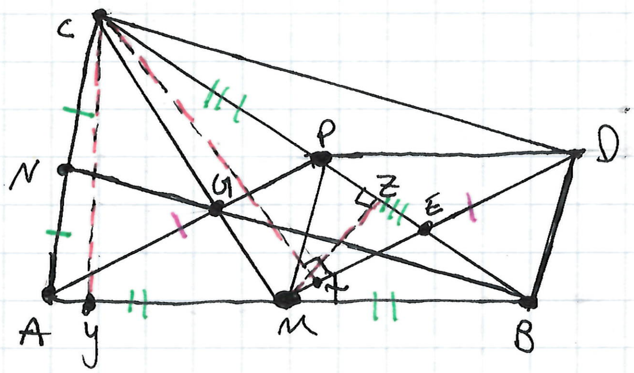

Given: M, N, P are midpoints of AB, AC, BC respectively

MD // AP

MD = AP

a.) Prove Area($\triangle$CMD) = 2 Area($\triangle$CME).

By hw8 #3, PDBM is a parallelogram.

By theorem 14, PE = EB & ME = ED.

By BF6, MD = ME + ED. Since ME = ED, ~~AN~~ By algebra, (*) MD = 2ME.

By BF12, draw a line through C, $\perp$ to MD with the point of intersection X.

By Thm 7, Area($\triangle$CMD) = 1/2 MD $\cdot$ CX & Similarly, (1) Area($\triangle$CME) = 1/2 ME $\cdot$ CX

By (*) & Algebra, (2) Area($\triangle$CMD) = 1/2 $\cdot$ 2 ME $\cdot$ CX.

[redacted]

By combining (1) & (2), we get Area($\triangle$CMD) = 2 $\cdot$ Area($\triangle$CME), as claimed

dLC

b.) Prove Area($\triangle$ABC) = 2 $\cdot$ Area($\triangle$CMB)

By BF12, draw a line through C $\perp$ to AB and have the point of intersection Y.

By thm 7, (3) Area($\triangle$ABC) = 1/2 AB $\cdot$ CY & Similarly, (4) Area($\triangle$CMB) = 1/2 MB $\cdot$ CY

By given, M is the midpoint of AB, and by definition, AM = MB. By BF6, AB = AM + ~~AM~~ MB, and by algebra, AB = 2MB. (**)

By combining (3) & (**), (5) Area($\triangle$ABC) = 1/2 $\cdot$ 2 MB $\cdot$ CY.

By combining (4) & (5), Area($\triangle$ABC) = 2 $\cdot$ Area($\triangle$CMB), as claimed

dLC

C.) Prove Area $(\triangle CME) = \frac{3}{4}$ Area $(\triangle CMB)$.

By part a.), $PE = EB$ (***). By given, P is the midpoint of BC and by defn. $PC = PB$.

By BF6, (1) $BC = PC + PB$ & similarly, $PB = PE + EB$
Since $PE = EB$, & $PC = PB$, (2) $PB = 2EB = PC$.

By combining this and (1), $BC = 4EB$.

By BF6, $CE = PC + PE$, but by (2) & (***), $CE = 3EB$.

$BC = CE + EB \underset{\text{By combining}}{\Rightarrow} BC = 3EB + EB \overset{\text{by algebra}}{\Rightarrow} \frac{3}{4}BC = CE$.

By BF12, draw a line through M $\perp$ to BC and have the point of intersection be Z.

By thm 7, (3) Area $(\triangle CME) = \frac{1}{2} MZ \cdot CE$ & similarly,

(4) Area $(\triangle CMB) = \boxed{\frac{1}{2} MZ \cdot BC.}$

We know $\frac{3}{4}BC = CE$, by combining this and (3)

(5) Area $(\triangle CME) = \boxed{\frac{1}{2} MZ} \cdot \frac{3}{4} \boxed{BC.}$

By combining (4) & (5), Area $(\triangle CME) = \frac{3}{4}$ Area $(\triangle CMB)$, as claimed

dLC