September 14, 2019

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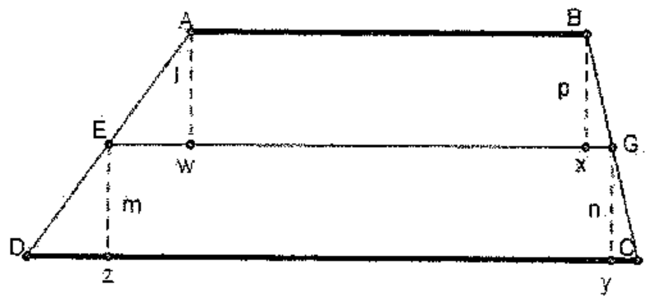
Homework 3: #9

Given: ABCD is a **trapezoid** in which AB||DC, E is the midpoint of AD and EG||AB.

To prove: G is the midpoint of BC.




Proof:

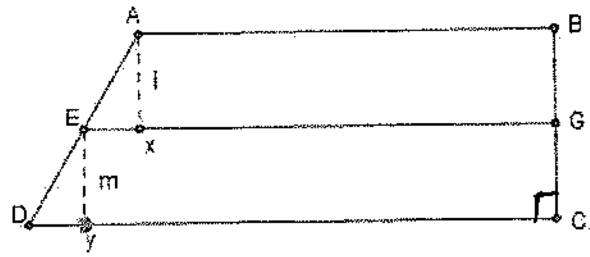
Case (i):

It is important to note that we know that ABCD is a trapezoid. From lecture 1, day 1 of MA 460, the definition of trapezoid is a quadrilateral with **exactly one pair of parallel sides**.

Let 'ℓ' be a line through A ⊥ EG and let 'p' be a line through B ⊥ EG. Likewise, let lines 'm' and 'n' be lines through points E and G that are both perpendicular to line DC (BF 7 allows us to construct lines while BF 12 allows us to specifically construct perpendicular lines). Also let w, x, y, and z be the points of the perpendicular intersections.

Given that AB||DC and AB||EG, then by BF 14, AB||EG||DC.
Given that E is the midpoint of AD, then by definition, EA = DE.
By definition of a perpendicular lines, ∡AWE = ∡EZD = ∡BXG = ∡GYC = 90 degrees.
By BF 5, ∡AEW = ∡EDZ and ∡BGX = ∡GCY.
By Theorem 3, ∡AEW + ∡AWE + ∡EAW = 180 degrees.
Since ∡AEW = ∡EDZ (BF 5) and ∡AWE = ∡EZD (definition of perpendicular lines), then by Theorem 4, ∡EAW = ∡DEZ.
Because ∡EDZ =∡AEW (BF 5), DE = EA (definition of midpoint), and ∡EAW = ∡DEZ (Thm 4), then by BF 3△ EDZ ≅△ AEW.
By definition of congruence ℓ = m.
By Theorem 15, ℓ = p, and m = n.
By Thm 15, definition of congruence, and by algebra, ℓ = m = p = n.
Because ∡BXG = ∡GYC (by definition of perpendicular lines) and ∡BGX = ∡GCY (by BF 5), then by Theorem 4, ∡XBG =∡YGC.
Because ∡BXG = ∡GYC (definition of perpendicular lines), p = n (Thm 15), and ∡XBG = ∡YGC (Thm 4), then by BF 3△ BGX ≅△ GCY.
By definition of congruence, BG = GC.
By BF 6, BG + GC = BC.
By Theorem 16 (a) and by algebra, BC = 2BG = 2GC, which means G is the midpoint of BC, as claimed. ☆

Case (ii): The example does not specifically say that there are no perpendicular lines in the trapezoid, so we must assume this possibility.




Proof:

Let ABCD be a trapezoid with BC⊥DC.

Let 'ℓ' be a line through A⊥EG and let 'm' be a line through E⊥DC. Let x be the point where 'ℓ' and EG intersect and let y be the point where 'm' and DC intersect (BF 7 allows us to construct lines while BF 12 allows us to specifically construct perpendicular lines).

Given that AB||DC and AB||EG, then by BF 14, AB||EG||DC.

Given that E is the midpoint of AD, then by definition, EA = DE.

By definition of perpendicular lines ∡AXE = ∡EYD = 90 degrees.

By BF5, ∡AEX = ∡EDY.

By Theorem 3, ∡AXE + ∡AEX + ∡EAX = 180.

Since ∡AEX = ∡EDY (BF 5) and ∡AXE = ∡EYD (by definition of perpendicular lines), then by Theorem 4, ∡EAX = ∡DEY.

Because ∡EDY =∡AEX (BF 5), DE = EA (definition of midpoint), and ∡EAX = ∡DEY (Thm 4), then by BF 3△ EDY≅△ AEX.

By definition of congruence ℓ = m.

By Theorem 15, ℓ = BG and m = GC.

By algebra, ℓ = m = BG = GC.

By BF 6, BG +GC = BC.

By Theorem 16 (a) and by algebra, BC = 2BG = 2GC, which means G is the midpoint of BC, as claimed. ☆