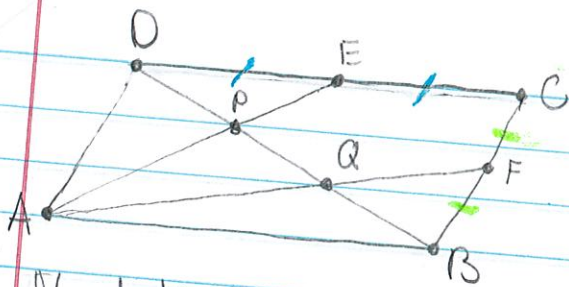


HW 4 Problem 4



Given:
 ABCD is a parallelogram
 E is midpoint of CD
 F is midpoint of CB

Need to prove $OP = PQ = QB$.

Thm 26:

$AB \parallel CD$ & $DB \rightarrow \angle PDE = \angle PBA$

$AB \parallel CD$ & $AE \rightarrow \angle DEP = \angle BAP$

Thm 11b) $\rightarrow \angle EPD = \angle APB$

def of similar triangles $\rightarrow \triangle DEP \sim \triangle BAP$

a similar argument can be made for $\triangle BFG \sim \triangle DAQ$

with $AD \parallel BC$ and DB & AF

Thm 11 & def of midpoint & similarities:

$$DE = \frac{1}{2} CD \rightarrow DE = \frac{1}{2} BA$$

$$BF = \frac{1}{2} BC \rightarrow BF = \frac{1}{2} DA$$

Thm 20:

$$OP = \frac{1}{2} BP \quad (1)$$

$$BQ = \frac{1}{2} OQ \quad (2)$$

BFG:

$$BP = PQ + QB \quad (3)$$

$$OQ = PQ + OP \quad (4)$$

Some algebra: (1) & (3), (2) & (4)

$$2OP = PQ + QB \rightarrow QB = 2OP - PQ \quad (5)$$

$$2BQ = PQ + OP \quad (6)$$

Combine 5 & 6

$$4OP - 2PQ = PQ + OP$$

$$3OP = 3PQ$$

$$OP = PQ \quad (8)$$

Combine (1), (3), & (8)

$$BP = \frac{1}{2} BP + QB$$

$$OP = \frac{1}{2} BP = QB$$

$$OP = PQ = QB$$