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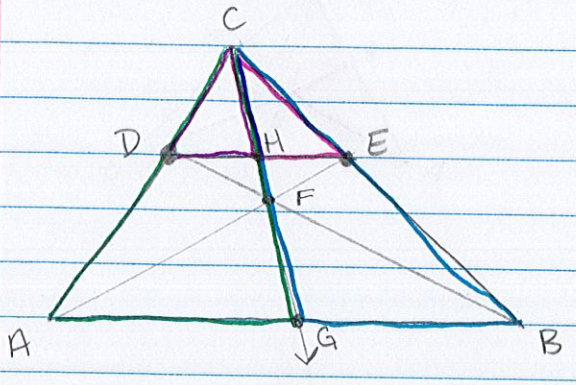
MA 4160

Homework 4 Problem 5



Given: Let D and E be points on AC and BC respectively. $DE \parallel AB$ F is the intersection of the segments DB and AE. Let G be the intersection of AB with the ray CF and H be the intersection of DE with the ray CF.

Prove: (a) $\frac{AG}{DH} = \frac{BG}{EH}$

(b) $\frac{AG}{EH} = \frac{BG}{DH}$

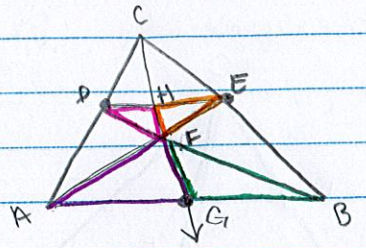
(c) G is the midpoint of AB

(a) By HW4 Problem 3, $\triangle CDH \sim \triangle CAG$ and $\triangle CEH \sim \triangle CGB$

By BFH $\frac{AG}{DH} = \frac{CG}{CH}$ and $\frac{BG}{EH} = \frac{CG}{CH}$

By algebra, $\frac{AG}{DH} = \frac{BG}{DH}$ as claimed ///

(b) prove $\frac{AG}{EH} = \frac{BG}{DH}$



By Theorem C

$\Delta AFG \sim \Delta EFH$

By Theorem C

$\Delta BGF \sim \Delta DHF$

By BF 4, $\frac{AG}{EH} = \frac{FG}{FH}$ and $\frac{BG}{DH} = \frac{FG}{FH}$

by algebra, $\frac{AG}{EH} = \frac{BG}{DH}$ as claimed ///

(c) Prove G is a midpoint of AB

→ using parts (a) and (b)

by algebra, (a) $(AG)(EH) = (BG)(DH)$

Solve for EH $EH = \frac{(BG)(DH)}{AG}$

same for (b) $(AG)(DH) = (BG)(EH)$

$EH = \frac{(AG)(DH)}{BG}$

by algebra, $\frac{(BG)(DH)}{AG} = \frac{(AG)(DH)}{BG}$

$(BG)^2 \cancel{(DH)} = (AG)^2 \cancel{(DH)}$

$(BG)^2 = (AG)^2$

$BG = AG$

by def of midpoint, G is the midpoint of AB as claimed ///