Math 460: Homework # 11. Due November 19

1. (Use Geometer's Sketchpad.) Begin with a point A and four lines ℓ, m, n and p that go through A. Next, hide the points other than A used to construct these lines (this is important). Choose a point B on ℓ and a point C on m, and let D and E be the intersections of $\overleftrightarrow{BC}$ with n and p respectively. Find a combination of the distances BC, CD, BE and DE which doesn't change when the points B and C are moved (leaving A and the lines ℓ, m, n and p fixed). Hint: the combination is the product of two of the lengths divided by the product of the two others.

2. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 10). See Figure 1. Given: $\angle 1 = \angle 2$, $\angle 3 = \angle 4$, $\angle 5 = \angle 6$, $\angle 7 = \angle 8$. To prove: $AB + CD = BC + AD$.

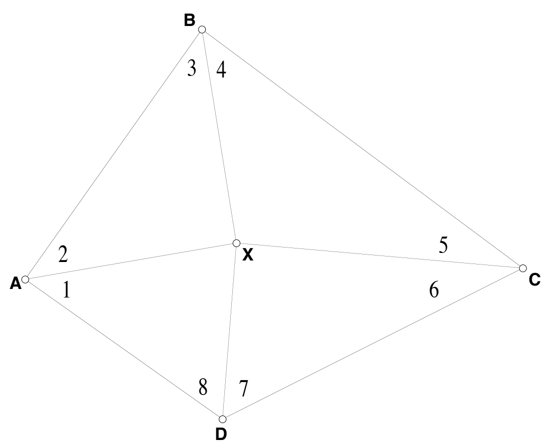

Figure 1

3. Prove Theorem 37 in the text. (Hint: Use a strategy similar to that of Theorem 35.)

4. Given: the line through A and B contains the points C and D. Let ℓ be the line through A and B and let m be the line through C and D. To prove: ℓ and m are the same line. (Hint: Use a Basic Fact.)

5. Euclid's proof of Proposition 23 uses Proposition 8 as one ingredient. Find a new proof of Proposition 23 that doesn't use Proposition 8. For this purpose you are allowed to use anything that comes before Proposition 8, and also Proposition 11, since we proved Proposition 11 without using Proposition 8.

6. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 8.) See Figure 2. Given: nothing. To prove:

$$\frac{BC}{BE}\frac{DE}{CD} = \frac{\sin(\angle BAC)\sin(\angle DAE)}{\sin(\angle BAE)\sin(\angle CAD)}$$

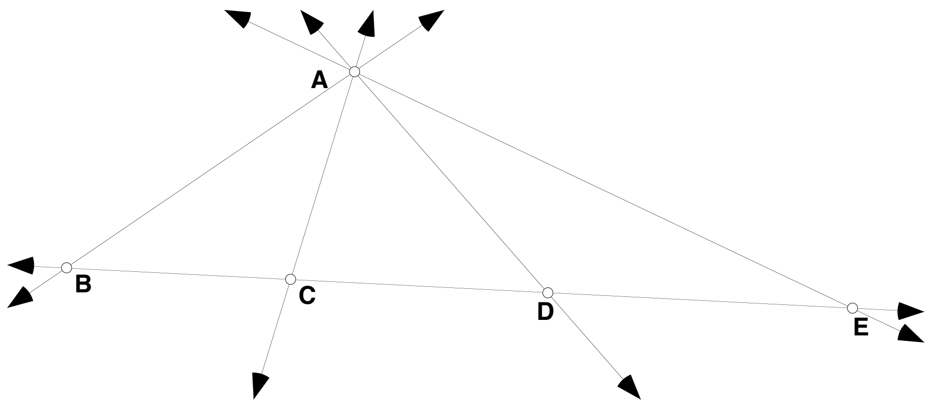

Figure 2

7. (See Figure 3) Given: Two triangles $\triangle ABC$ and $\triangle ABD$ which share the side AB, as shown. Let G be the centroid of $\triangle ABC$ and J be the centroid of $\triangle ABD$. Prove that GJ is parallel to CD, and that the length of GJ is $\frac{1}{3}CD$.

   (Hint: It will help to first prove a 'lemma'—i.e., a result that is proven separately, but is designed to help you with the more important proof. Here's a suggested lemma: Given a triangle $\triangle XYZ$, a number $0 < r < 1$, a point P on XY with $XP = r \cdot XY$, and a point Q on XZ with $XQ = r \cdot XZ$. Prove that PQ is parallel to YZ and $PQ = r \cdot YZ$.)

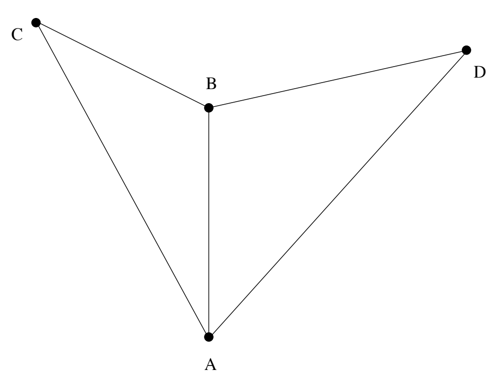

Figure 3

8. (See Figure 4). Given $ABCD$ is a trapezoid with AB parallel to DC, and DC longer than AB as shown. M, N, P, and Q are the midpoints of the indicated segments. (i) Prove that $PQ = \frac{1}{2}(DC - AB)$ (you may use the fact you proved on Assignment 10 for this situation).(ii) Find a similar formula for the length of MN and prove that your formula is correct.

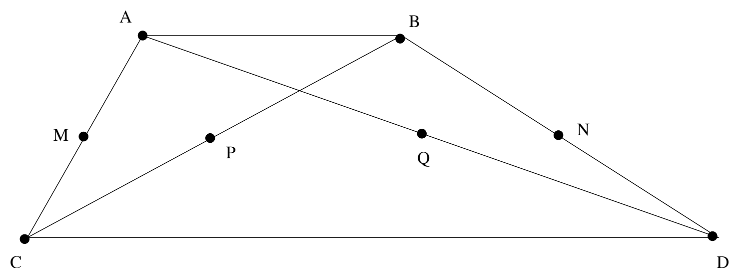

Figure 4

9. See Figure 5. Given: the lines that look straight are straight. To prove: BC is parallel to FG.

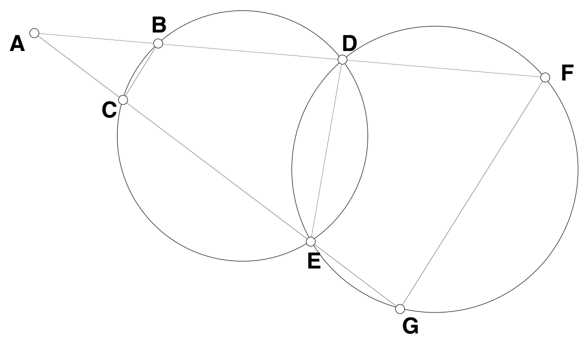

Figure 5

10. Prove that a parallelogram is a rectangle if and only if the diagonals are of equal length.

11. Let ABC be a triangle. Show there is a circle tangent to lines AB, AC, and BC which lies outside the triangle (See Figure 6). Make your own figure in Sketchpad, showing the critical elements used to construct this circle. Such a circle is said to be *escribed* to the triangle. Notice that there are three escribed circles to a given triangle, while there is only one inscribed triangle.

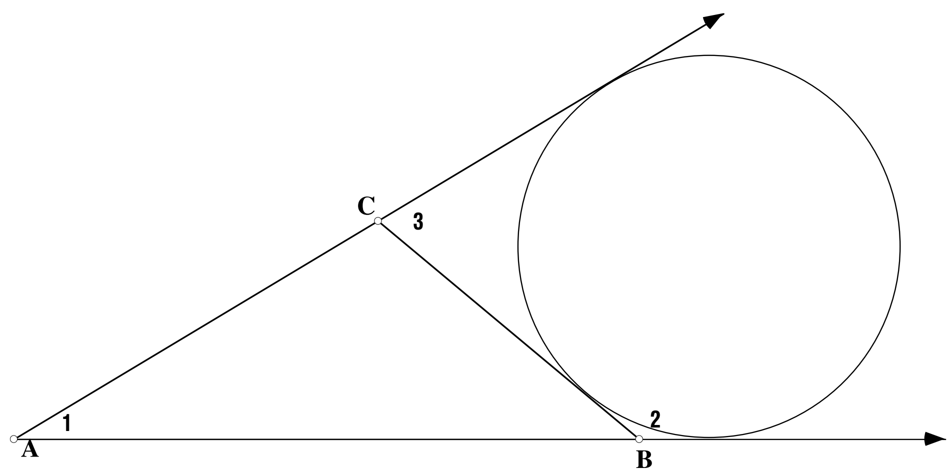

Figure 6

12. (See Figure 7.) If $AEGD$ and $DGFB$ are cylic quadrilaterals, prove that so is $GECF$.

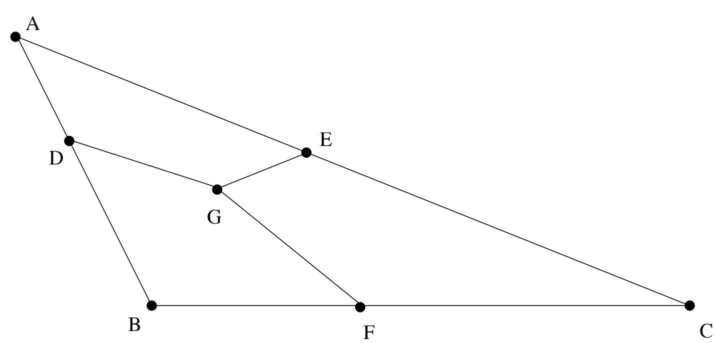

Figure 7