Math 460: Homework # 12 . Tuesday, November 26

1. (Use Geometer's Sketchpad.) In Figure 1, find an equation relating $\angle 1$, arc ABC and arc DEF.

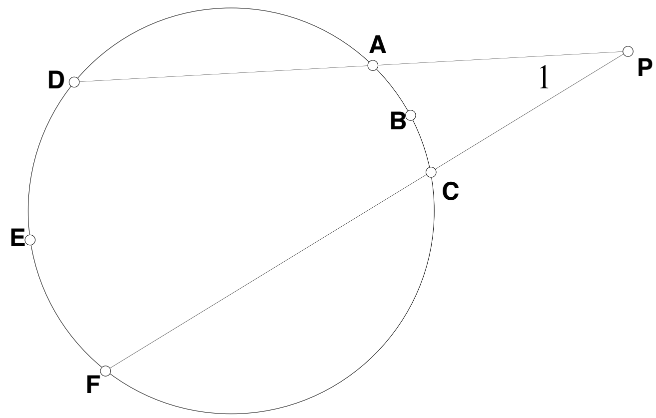

Figure 1

2. (Use Geometer's Sketchpad.) Construct a triangle ABC and its incenter I. Let ℓ, m and n be the perpendiculars from I to the three sides AB, AC and BC respectively. Let D be the intersection of ℓ with AB, let E be the intersecton of m with AC, and let F be the intersection of n with BC. (D, E and F are called the "feet" of the perpendiculars from I to the three sides.) Now hide the lines ℓ, m and n. Finally, draw in the lines AF, BE and CD. What do you notice about these three lines?

3. (Use Geometer's Sketchpad.) A quadrilateral $ABCD$ is called **incyclic** if a circle can be inscribed inside of it—meaning that the circle is tangent to all the sides (see Figure 2). By experimenting with Geometer's Sketchpad, determine a criterion in terms of lengths or angles for deciding if a quadrilateral is incyclic. Print out two pictures showing evidence for your conjecture, and include any pertinent measurements. (It might help, when you are first starting to work on this, to look for a criterion that is similar to the criteria we know for identifying cyclic quadrilaterals.)

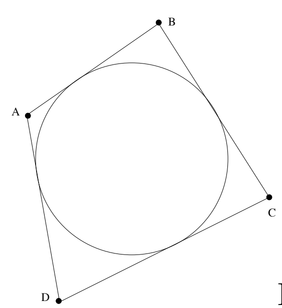

Figure 2

4. (See Figure 3.) Given: $\angle 1 = \angle 2$. To prove: $\angle ABC + \angle ADC = \angle BCD + \angle BAD$.

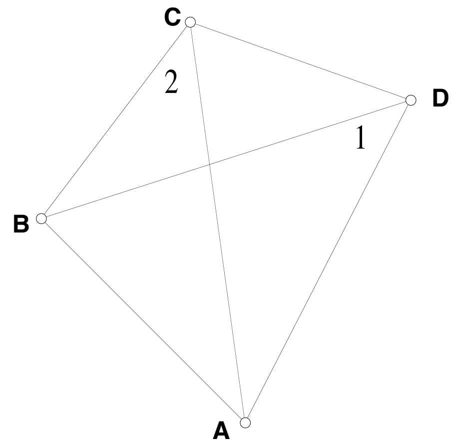

Figure 3

5. (See Figure 4) Let $\mathcal{C}$ be a circle with center O and suppose A is on $\mathcal{C}$. Let P be any point other than O. Prove that the circle $\mathcal{K}$ with **diameter** OP contains the midpoint M of any segment AB such that B is on $\mathcal{C}$ and the line AB contains P. **Hint:** Use what you know about M to show that $\triangle POM$ is a right triangle. Then use what you know about the center of the circumcenter of a right triangle.

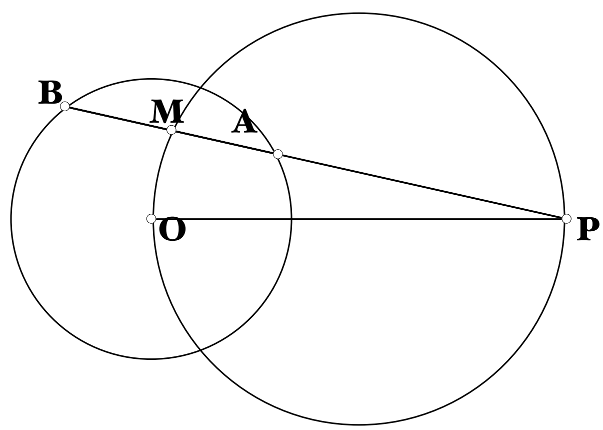

Figure 4

6. (See Figure 5.) Given: M and N are midpoints, $AC = DF$, $MC = NF$, $BC = EF$. To prove: $\triangle ABC \cong \triangle DEF$. (Hint: draw in one extra line in each triangle.)

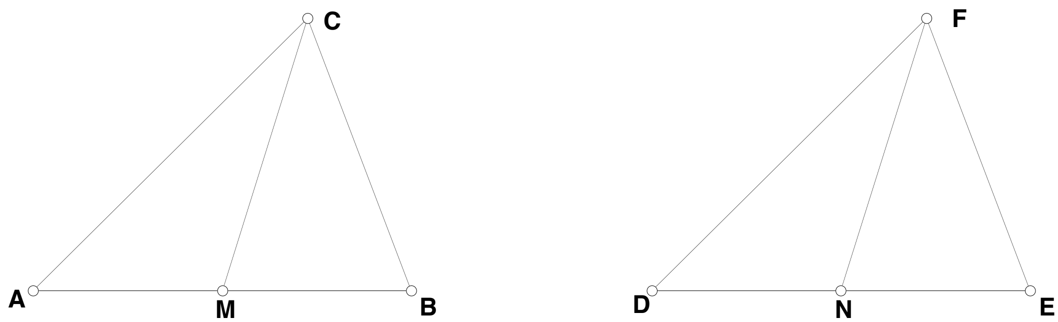

Figure 5

7. In Euclid's proof of Proposition 8, lines 13–17 use the method of "applying" one triangle to another. Find a way to replace lines 13–17 by an argument that doesn't use the idea of "applying." You may use anything that comes before Proposition 8, and you may also use Proposition 23 (since we have proved Proposition 23 without using Proposition 8.)

8. Now discover a different proof of Euclid Proposition 8 by contemplating Figure 7. You may use anything that comes before Proposition 6, and also Proposition 23. Do not use the method of "applying" one triangle to another, and do not use proof by contradiction. There are three cases, but you will get partial credit for doing one case.

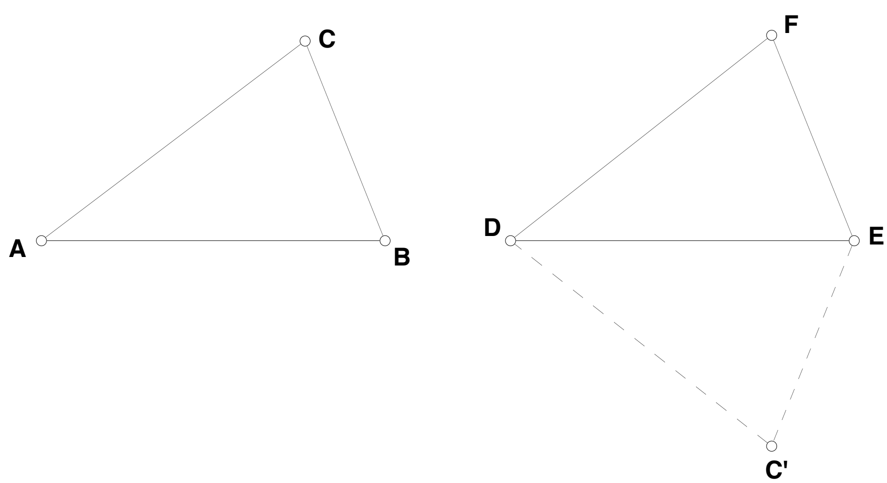

Figure 6

9. (See Figure 7.) Given: M is the midpoint of AB, and the lines that look concurrent are concurrent. To prove: DE is parallel to AB. (Hint: use Theorem 36 as one ingredient.)

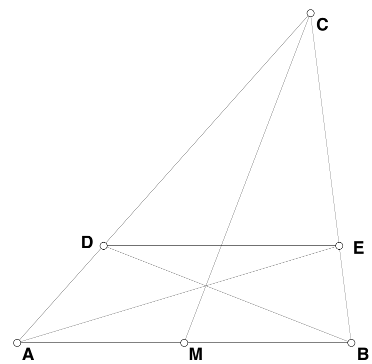

Figure 7

10. Prove the converse of the Pythagorean Theorem: If $\triangle ABC$ is a triangle in which $AB^2 + BC^2 = AC^2$, then $\angle B$ is a right angle. Euclid does this one way in Proposition 48, but here we will give an alternate approach that uses proof by contradiction. First assume that $\angle B < 90°$, and deduce a contradiction by contemplating the picture below. Then do the case where $\angle B > 90°$ by a similar method.

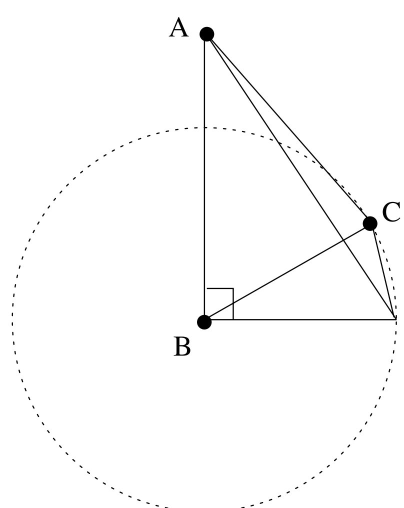

Figure 8

11. (In this problem we finish—at last—the proof of the fact that you discovered in Problem 2 of Assignment 7.) See Figure 1. Given: G is the centroid of $\triangle ABC$ and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the area of hexagon $UVWXYZ$ is exactly 13/36 of the area of $\triangle ABC$. (This is pretty easy if you combine facts that you proved in earlier homework.)

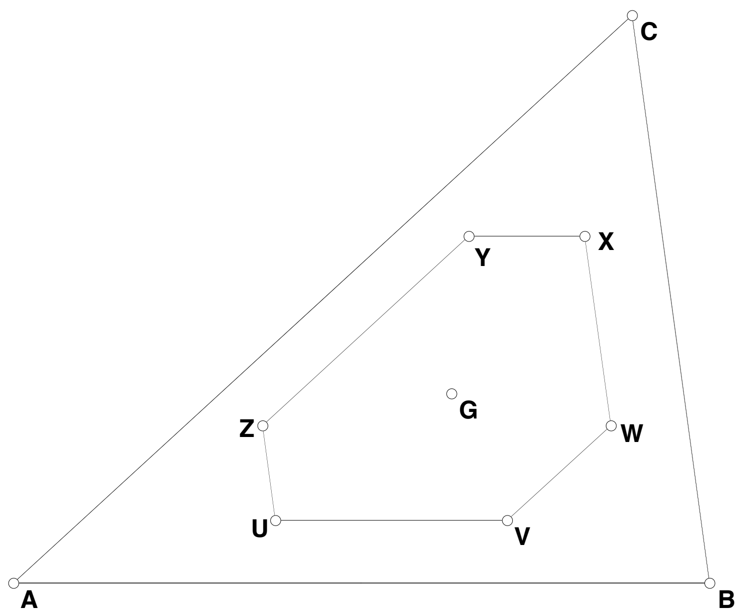

Figure 9