Math 460: Homework # 13. Due Tuesday, December 2

1. (In this problem we prove the fact that you discovered in Problem 1 of Assignment 12.) In Figure 1, prove that $\angle 1 = \frac{1}{2}\text{arc } DEF - \frac{1}{2}\text{arc } ABC$.

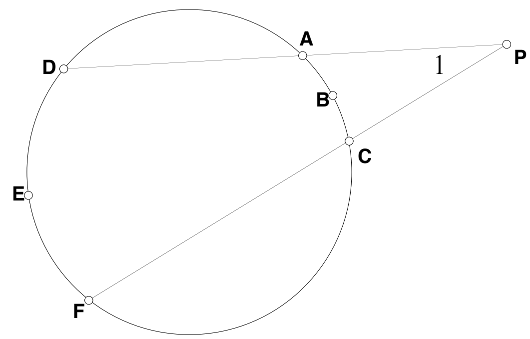

Figure 1

2. (In this problem we prove the fact that you discovered in Problem 2 of Assignment 12.) See Figure 2. Given: I is the incenter of $\triangle ABC$ and the lines ID, IE and IF are perpendicular to AB, AC and BC respectively. To prove: the lines AF, BE and CD are concurrent. (Hint: use Theorem 37.)

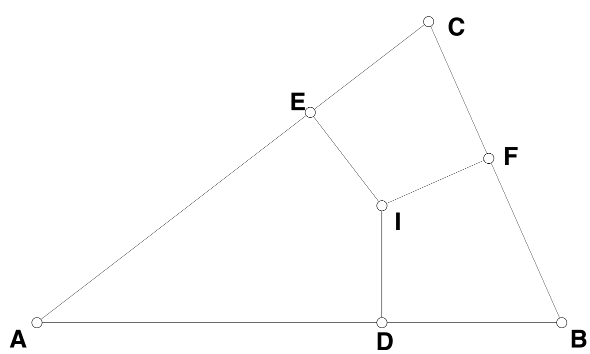

Figure 2

3. Prove the $\Longrightarrow$ direction of Theorem 43 of the course notes, using only the definition of tangent and facts from Euclid. (Hint: proof by contradiction; see Figure 3. If m is not perpendicular to OA, show that it is possible to construct a second point A' on m with OA=OA'.)

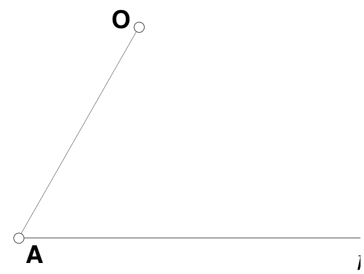

Figure 3

4. Here we prove the fact you discovered in problem 3 of Homework 12. A quadrilateral $ABCD$ is called **incyclic** if a circle can be inscribed inside of it—meaning that the circle is tangent to all the sides (see Figure 4). Prove that if $ABCD$ is incyclic, then $AB - BC = AD - DC$.

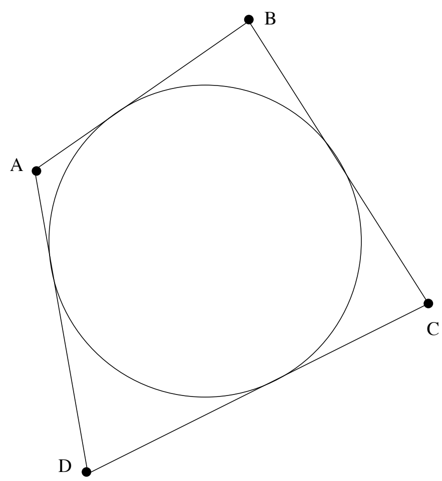

Figure 4

5. Given: Two parallel lines l and m, and a transversal k. Prove that there is a circle which is tangent to all three lines. (Hint: Think about how we constructed the inscribed and escribed circles of a triangle).

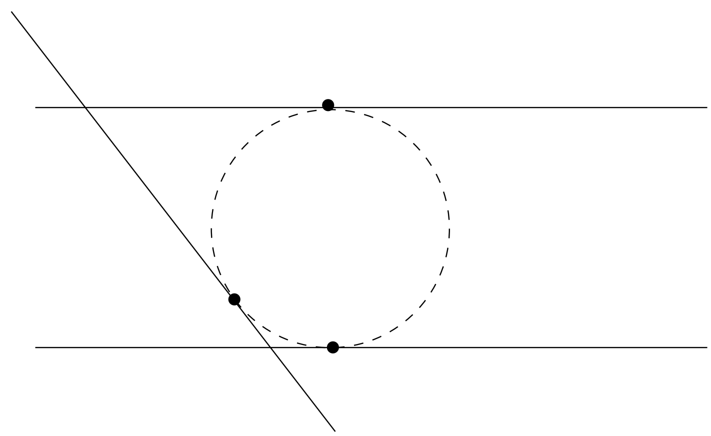

Figure 5

6. (See Figure 9) Given: H is the orthocenter of $\triangle ABC$; P, Q, and R are the midpoints of AB, AC, and BC; U, V, and W are the midpoints of AH, CH, and BH. Prove that P, Q, R, U, V, and W all lie on a common circle. (Hint: Draw the circle through three of the points—you will have to figure out which ones to choose—and then show that the other points lie on this circle. Use cyclic quadrilaterals.)

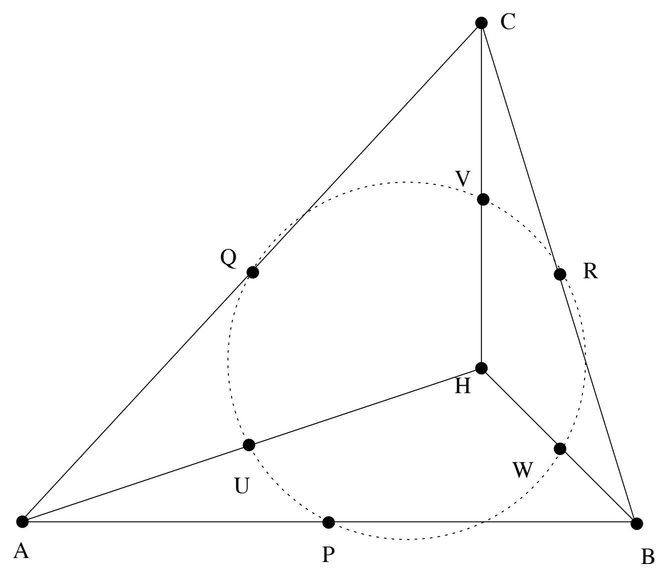

Figure 6