Math 460: Homework # 3. Due Thursday, September 5

1. Use Geometer's Sketchpad to construct a triangle, along with the following
   (a) its circumcenter (labeled O)
   (b) its incenter (labeled I)
   (c) its orthocenter (labeled H)
   (d) its centroid (labeled G)
   (e) the line through O and H.

   Hide all the lines used in constructions (a)-(d). Print out a copy, then change the shape of the triangle and print another copy. The line through O and H has a special property that should be obvious from your pictures–what is it? (You do not need to prove anything for this problem.)

2. (Use Geometer's Sketchpad) Start with a triangle ABC. Let D and E be points on the segments AC and BC, respectively, with DE parallel to AB. Let F be the intersection of the segments DB and AE, and let G be the intersection of AB with the ray CF. What special property does G have? Display a measurement which shows it has this property. Print the picture, then change the shape of the triangle, check that G still has this property, and print the new picture. You do not have to prove anything for this problem.

3. Give a proof of Theorem 13.

4. Give a proof of Theorem 14.

5. Let $ABCD$ be a parallelogram, and let M and N be the midpoints of AB and CD. Prove that MN is parallel to BC.

6. (See Figure 4.) Given $AE \perp BC$, $BD \perp AC$, and $AF = BF$, prove that $AC = BC$.

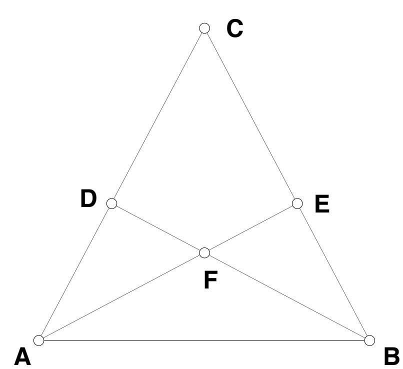

Figure 4

7. (See Figure 5) Given $MK = MQ, \angle K = \angle Q$, PM is perpendicular to MK, and LM is perpendicular to MQ, prove $RS = TS$. (Hint: Use what was shown in problem 7 of the last assignment.) **Warning:** Although it is true that equals subtracted from equals give equals, the same idea is *not* valid for congruence (it is not always true that congruent triangles subtracted from congruent triangles give congruent triangles.)

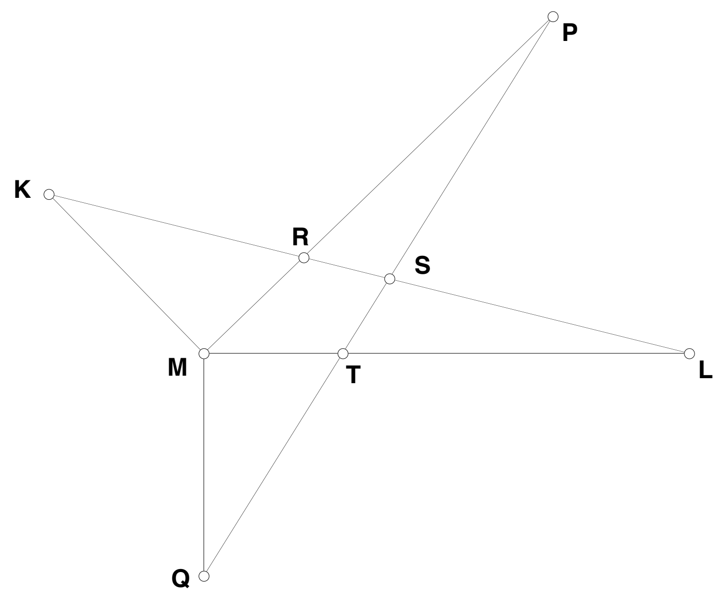

Figure 5

8. Given a quadrilateral $ABCD$ with $AB = BC$ and $CD = AD$, prove that the diagonals AC and BD are perpendicular.

9. Given: $ABCD$ is a trapezoid in which AB is parallel to DC, E is the midpoint of AD, and EG is parallel to AB. To prove: G is the midpoint of BC.

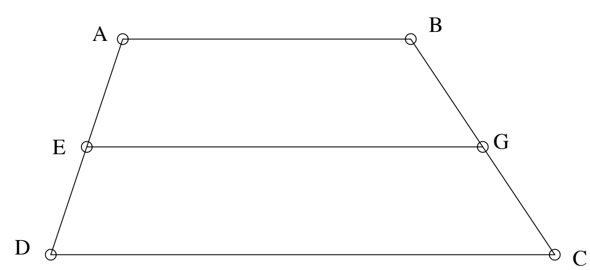

Figure 6

10. See Figure 7.Let ABC be a triangle and let $A'BC$ and ABC' be equilateral triangles. Show $AA' = CC'$.

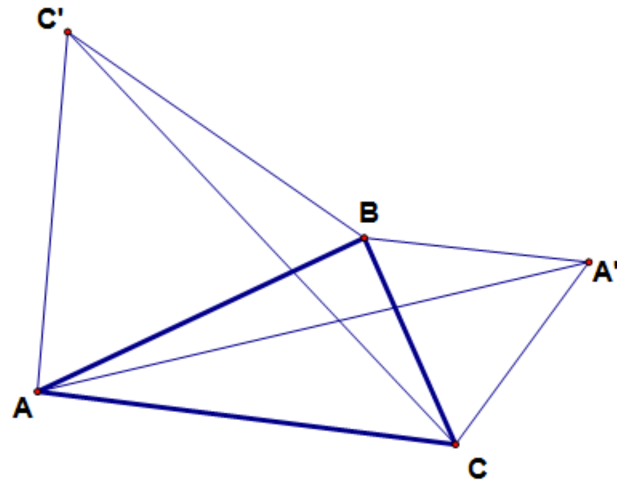

Figure 7

11. (See Figure 8) Given: DE is parallel to AB, EF is parallel to BC, and DF is parallel to AC. To prove: $\triangle ABC$ is similar to $\triangle DEF$.

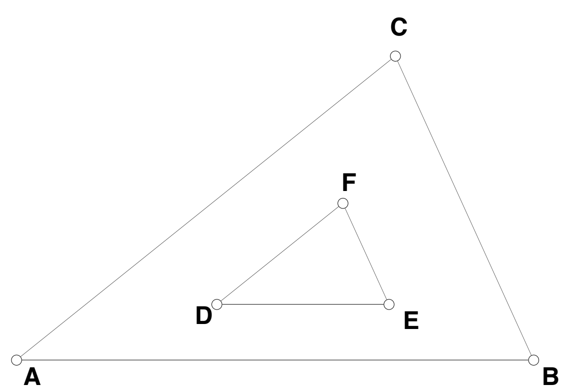

Figure 8

12. Use Figure 9. Assume the lines that look straight are. Let $ABCD$ be a quadrilateral. Given AH, BH, CF and DF are the angle bisectors of $ABCD$, prove $\angle FEH + \angle FGH = 180°$.

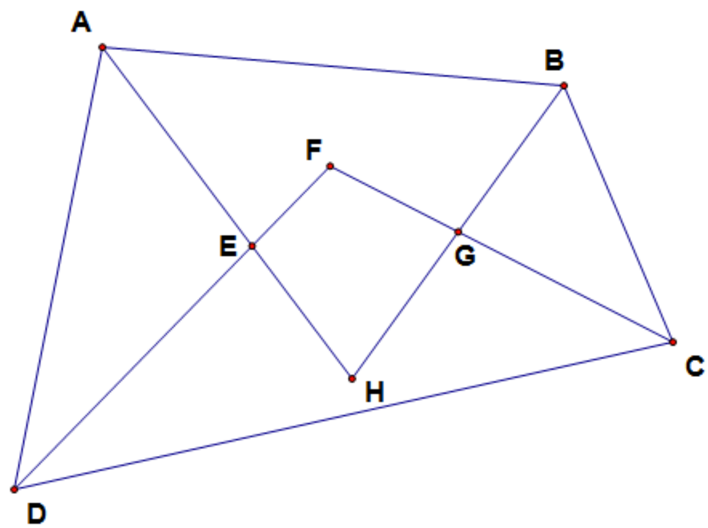

Figure 9