**Math 460: Homework # 5. Due Thursday September 26**

**Note:** Problems 1, 2, and 4 all use Geometer's Sketchpad.

1. (Use Geometer's Sketchpad.) Construct a quadrilateral $ABCD$ and let $M, N, P$ and $Q$ be the midpoints of the sides. Find the areas of the quadrilaterals $ABCD$, $MNPQ$ and of the four small triangles at the corners of $ABCD$ (use "Polygon interior" from the "Construct" menu and then "Area" from the "Measure" menu). Find an equation relating the areas of the four triangles. Then find an equation relating the areas of the two quadrilaterals. Print out a picture with the calculations that demonstrate the equations you found. You do not have to prove anything for this problem.

2. (Use Geometer's Sketchpad.) Draw a triangle $ABC$ and a point $P$ in the interior of $ABC$. Draw the lines (not just the line segments) connecting $P$ to each of the vertices, and let $D, E$, and $F$ be the points where these lines meet $AB$, $AC$ and $BC$, respectively. Find an equation relating the three ratios $\frac{DA}{DB}$, $\frac{EC}{EA}$ and $\frac{FB}{FC}$. Print out a copy of your picture, including the measurements and calculations.

3. (See Figure 1.) Give the proof of Theorem 24 for Case (iii). Given: $M$ and $N$ are the midpoints of $AB$ and $AC$, $MX \perp AB$, $NX \perp AC$, and $X$ is on $BC$. To prove: $X$ is on the perpendicular bisector of $BC$.

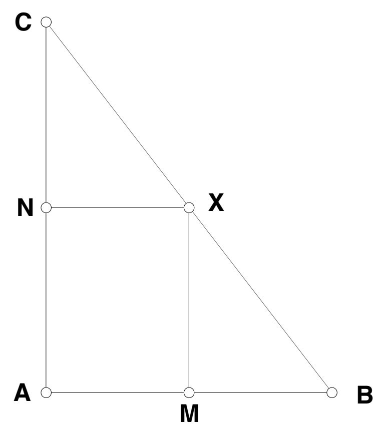


Figure 1

4. Prove Theorem 26. (Hint: Use ideas similar to those of the proof of Theorem 24, but note that there is only one case.) Make a picture to illustrate your proof with Geometer's Sketchpad.

5. (See Figure 2.) Given: $AD = BC$, $AC = BD$, $AK = BN$. To prove: $KG = NH$.

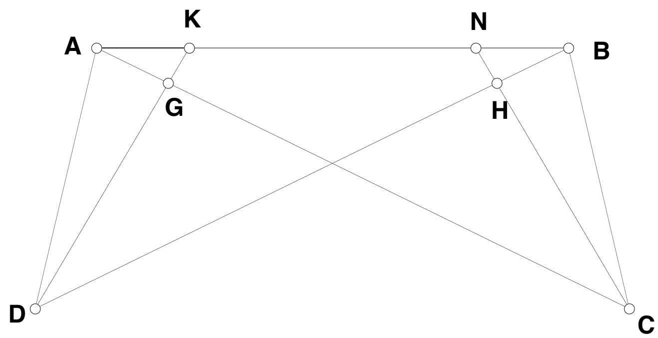


Figure 2

6. (See Figure 3.) Given: The lines that look straight are straight, $\angle D = \angle 1$, $KM = TM = CM$. To prove: $AD = BC$.

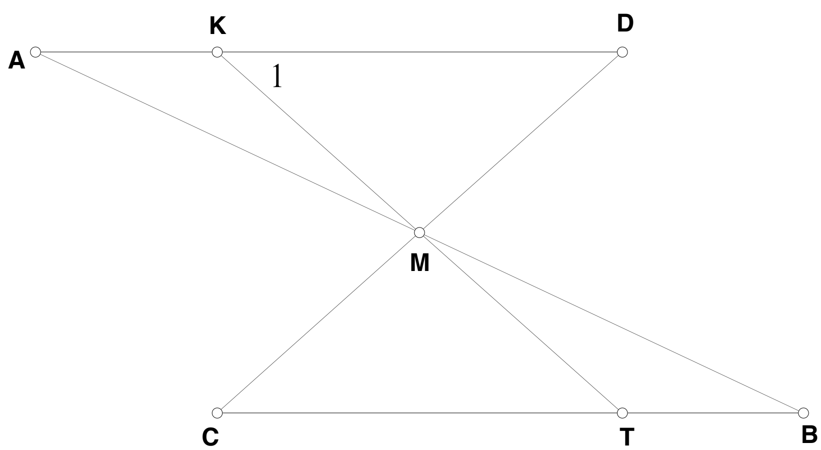


Figure 3

7. Here we prove the fact that was suggested in Problem 2 of Homework 4. Let $ABC$ be an equilateral triangle and let $P$ be a point which is outside the triangle but in the interior of $\angle ABC$. Let $a$, $b$, and $c$ be the distances from $P$ to $\overleftrightarrow{AB}$, $\overleftrightarrow{AC}$ and $\overleftrightarrow{BC}$ respectively. Let $h$ be the height of triangle $ABC$. To prove: $a - b + c = h$.

8. (See Figure 4.) Given: $DC$ is parallel to $EG$, and $EG$ is parallel to $AB$. To prove: $F$ is the midpoint of $EG$.

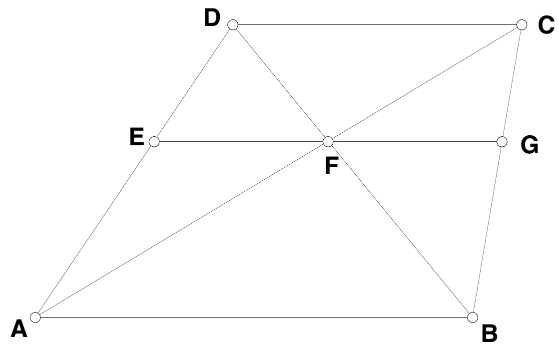


Figure 4

9. (See Figure 5.) Given: $\angle A = \angle B$, $AD = BE$, $\angle ADG = \angle BEF$. To prove: $\angle CFE = \angle CGD$.

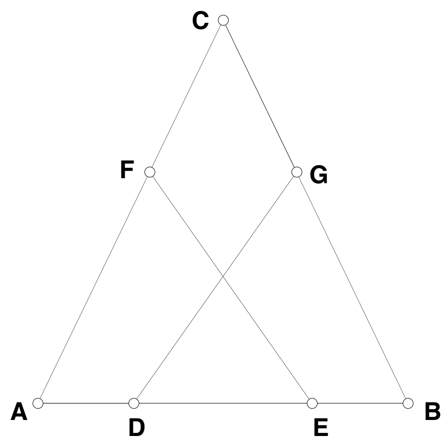


Figure 5

10. (See Figure 6) Let $ABCD$ be a parallelogram, $E$ a point on $AD$ and $F$ a point on $AB$ with $DE = BF$. Let $G$ be the intersection of $BE$ and $FG$. Prove $CG$ bisects $\angle BCD$.

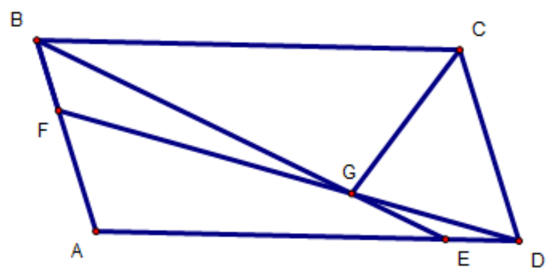


Figure 6