Math 460: Homework # 6. Due Thursday October 3

1. (Use Geometer's Sketchpad.) Draw a triangle ABC. Then draw a line ℓ through A parallel to BC, a line m through B parallel to AC, and a line n through C parallel to AB. Let D be the intersection of ℓ and m, E the intersection of ℓ and n, and F the intersection of m and n. Next, draw in the three altitudes of $\triangle ABC$ and the three perpendicular bisectors of $\triangle DEF$. What do you notice?

2. (Use Geometer's Sketchpad.) To say that a quadrilateral is *inscribed* in a circle means that all four of its vertices lie on the circle. Not every quadrilateral can be inscribed in a circle; when a quadrilateral can be inscribed in a circle its angles satisfy a certain equation. Find this equation and print out a copy of the picture with the calculations which demonstrates that the equation holds. You do not have to prove anything for this problem. (Note: for *every* quadrilateral it is true that the sum of the angles is $360°$, so this isn't the equation you're looking for.)

3. (In this problem we prove a fact that you demonstrated experimentally in Problem 1 of the fifth assignment.) Let $ABCD$ be a quadrilateral. Let M, N, P, and Q be the midpoints of the sides. Prove the area of $MNPQ$ is one half the area of $ABCD$.

4. (See Figure 1). Prove Case (ii) of Theorem 31. Given: A', B' and C' are collinear. To prove:
$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$

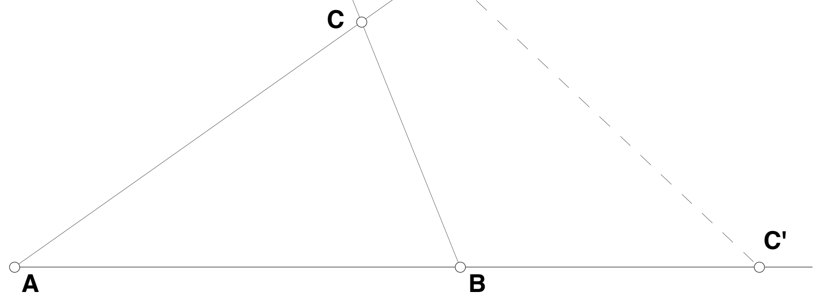

Figure 1

5. (See Figure 2.) Prove that the bisectors of angles 1, 2 and 3 are concurrent. (Hint: use a strategy similar to the proof of Theorem 26.) Make a picture to illustrate your proof with Geometer's Sketchpad.

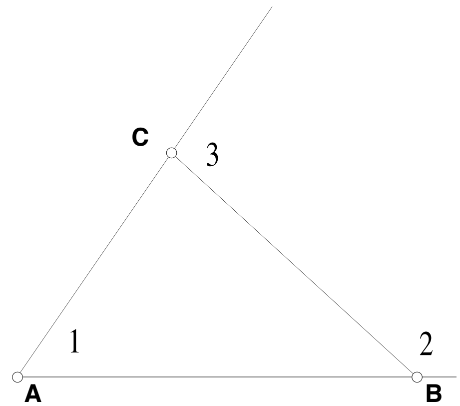

Figure 2

6. Suppose that you have a computer program which can perform the following functions:

   (a) It can draw points, and draw line segments connecting two points.

   (b) Given a point O and a line segment AB, it can construct the circle with center O and radius equal to the length of AB.

   (c) Given a line segment AB, it can find the midpoint.

   (d) Given a line l and a point P (not necessarily lying on l), it can construct the line through P which is perpendicular to l.

   Given a line l, a point A on l, and a point B not on l, devise a method for constructing a circle which passes through both A and B and is tangent to l. Make a list of the steps needed. Use Geometer's Sketchpad to demonstrate your method (print out a picture).

   (Hint: It might help to work backwards. Draw a rough sketch on paper showing what the circle should look like, then look at some of the theorems on our list about circles, particularly ones that give properties of of tangents. Also, look at Homework 1, problem 1. Then find some lines that you know the center of the circle is going to lie on.)

7. This is a continuation of problem #7. Given a circle C with center O, and a point P outside of C, devise a method for constructing a line through P which is tangent to the circle. Make a picture with Geometer's Sketchpad demonstrating your method. (You might look at Problem 3 of assignment 5, particularly the picture.)

   (You do not need to prove anything for problems #6 and #7).

8. (See Figure 3.) Given: $ABCD$ is a parallelogram, X is a point on the line AB, and Y is a point on the line BC. Prove that $\triangle DCX$ has the same area as $\triangle ADY$.

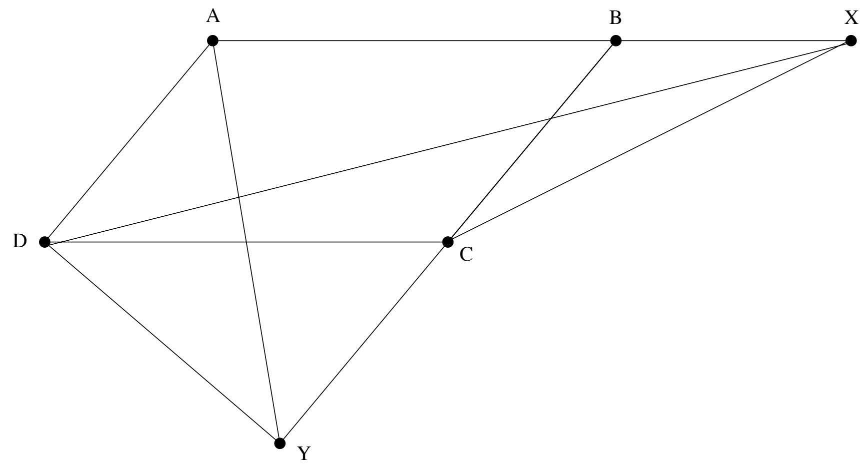

Figure 3

9. Let ABC be a triangle with centroid G. Let l be the line through G parallel to AB, and let D and E be the points where l intersects AC and BC respectively. Prove that the area of CDE is $4/9$ of the area of ABC.

10. (See Figure 4) Given: A quadrilateral $ABCD$ in which AB is parallel to DC, and E is the intersection of the diagonals. Show that if $AE = EC$ then $ABCD$ is a parallelogram.

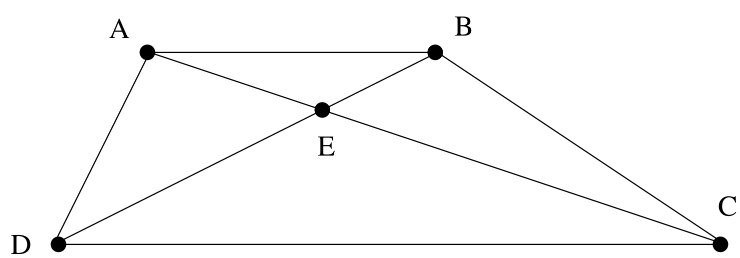

Figure 4