Math 460: Homework # 8. Due Thursday, October 17

1. (Use Geometer's Sketchpad.) Start with triangle ABC. Draw a line ℓ that crosses all three lines AB, AC and BC, and let P, Q and R be the intersections of ℓ with the lines AB, AC and BC respectively. Next draw another line m that crosses all three lines AB, AC and BC, and let P', Q' and R' be the intersections of m with AB, AC and BC respectively. Now let X be the intersection of BC and QP', Y the intersection of AC and PR', and Z the intersection of AB and RQ'. What do you notice about the points X, Y, and Z? Print out a picture, then change the position of the lines l and m and print another copy.

2. (Use Geometer's Sketchpad.)

   (a) Write a tool (or script) that constructs a square with a given line segment as its side. Print out a copy of this script.

   (b) Use the script from part (a) to make a picture like Figure 1. Here ABC is an arbitrary triangle, and the things that look like squares are squares.

   (c) Find an equation relating the areas of the shaded triangles. Then change the shape of triangle ABC and verify that this equation still holds.

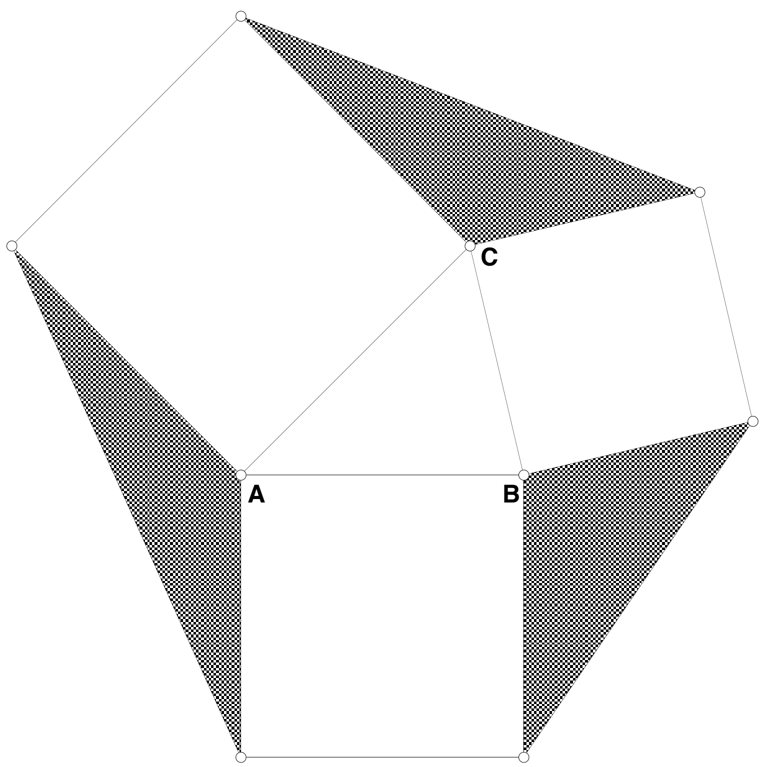

Figure 1

3. (See Figure 2.) Given: M, N and P are the midpoints of AB, AC and BC respectively, MD is parallel to AP, and $MD = AP$. To prove: $CD = NB$. (Hint: there are three parallelograms in this picture. Do *not* draw in any extra lines.)

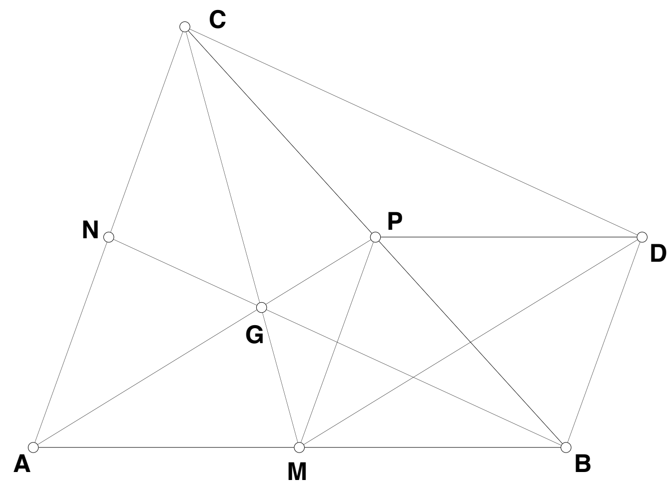

Figure 2

4. (See Figure 3.) Prove that

$$\frac{A'B}{A'C}\frac{B'C}{B'A}\frac{C'A}{C'B} = 1.$$

(Hint: Use a strategy similar to Problem 3 from Assignment 7.)

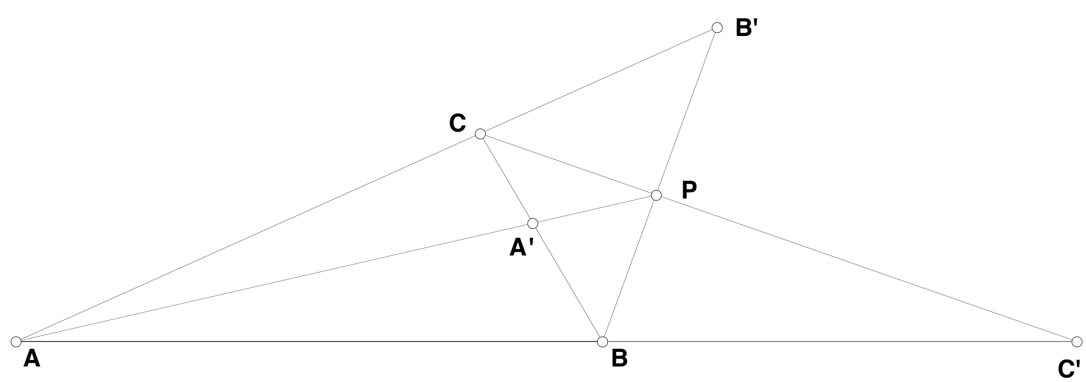

Figure 3

5. (See Figure 4.) Given: AB is parallel to DE, AC is parallel to DF, and BC is parallel to EF. To prove: the lines AD, BE and CF are concurrent.

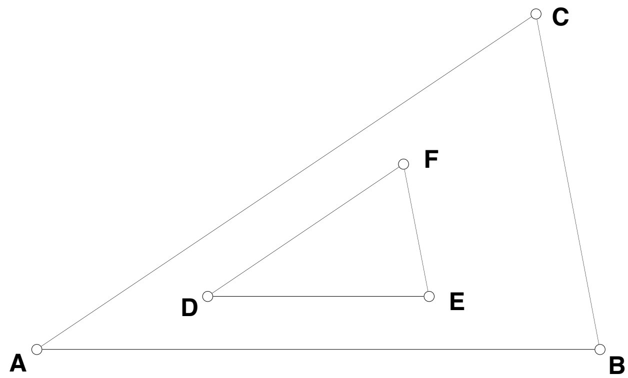

Figure 4

6. (In this problem we begin to prove the fact that you discovered in Problem 2 of Assignment 7.) See Figure 5. Given: M, N and P are the midpoints of AB, AC and BC respectively, and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: UV is parallel to AB.

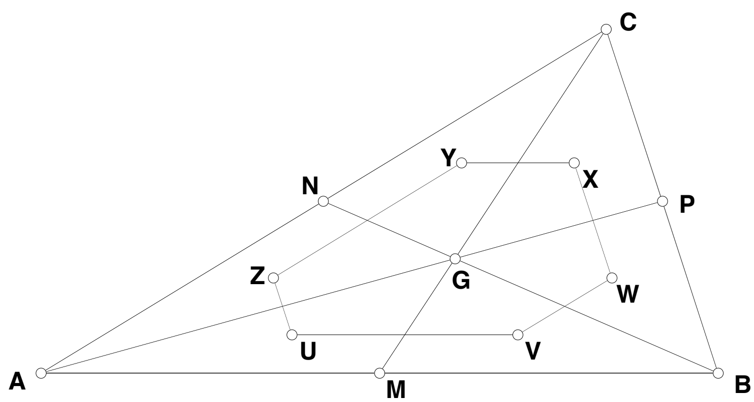

Figure 5

7. (See Figure 6.) Given $ABCD$ is a parallelogram, and BG is parallel to DH. To prove: $DF = BE$.

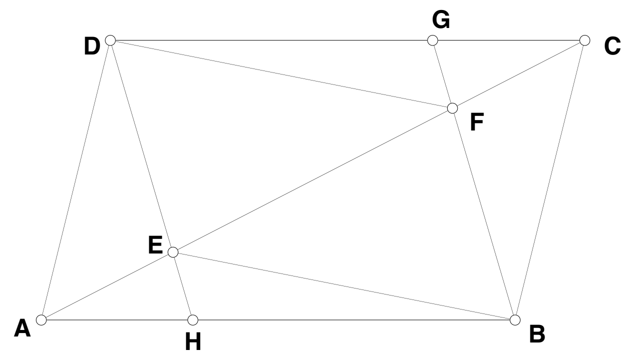

Figure 6

8. (10 points) (a) Given a segment AB and a point C (which may or may not be on AB). Prove:

$$C \text{ is on the perpendicular bisector of } AB \iff AC = BC.$$

(b) Use part (a) to give a shorter proof of Theorem 24. Begin as usual by constructing two perpendicular bisectors and their intersection X. Then connect X to the three vertices. After that you may not use any Basic Fact or Theorem.

9. A quadrilateral $ABCD$ is called a **cyclic quadrilateral** if the vertices A,B,C,D lie on a common circle.

   (See Figure 7) Given a cyclic quadrilateral $ABCD$, prove that

   (a) $\angle BAC = \angle BDC$ and $\angle ABD = \angle ACD$.

   (b) $\angle ADC + \angle ABC = 180° = \angle BAD + \angle BCD$.

   (Hint: The proof of (a) should be extremely short. Then (a) will help you do (b).)

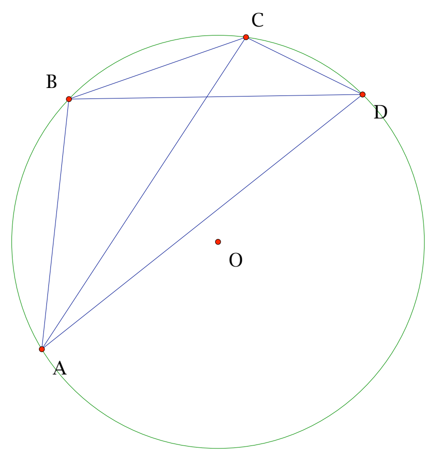

Figure 7

10. (See Figure 8.) Given: $AC = BC$ and $AD = BF$. To prove: $DE = EF$. Do not draw in any extra lines!

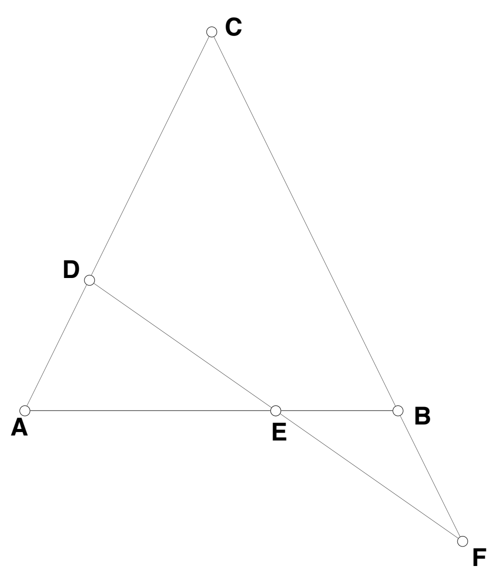

Figure 8

11. (See Figure 9.) Suppose that AB, XY are two different chords of a given circle, which intersect at a point E. Given that AB and XY bisect each other, prove that E must be the center of the circle. (Hint: Use proof by contradiction. Assume E is not the center of the circle, and draw in the center as point O. Connect O to E, and show that this line must be perpendicular to both AB and XY. Then use a Basic Fact to get a contradiction.)

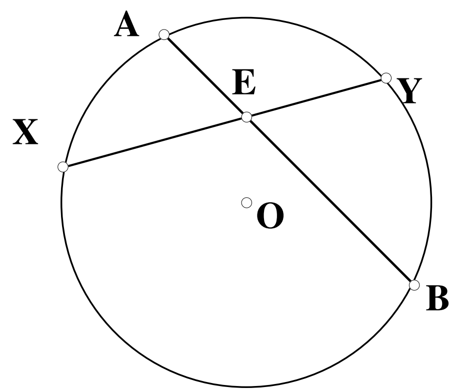

Figure 9