Math 460: Homework # 9. Due Thursday October 24

1. (Use Geometer's Sketchpad.)

   (a) Make a script which constructs the orthocenter of a given triangle. Print out a copy.

   (b) Construct a triangle ABC and play your script to find the orthocenter. Label the orthocenter H. Then play your script a second time, this time to find the orthocenter of triangle ABH. Indicate on your picture exactly where the orthocenter of triangle ABH is.

2. (Use Geometer's Sketchpad). Make a script that carries out the construction described in the proof of Euclid Proposition 11, using only the "Circle by center and radius" and "Segment" commands. Print out a copy.

3. (In this problem we prove what you discovered in Problem 2 of Assignment 8.) See Figure 1. Given: the things that look like squares are squares. To prove: the areas of the shaded triangles are all equal.

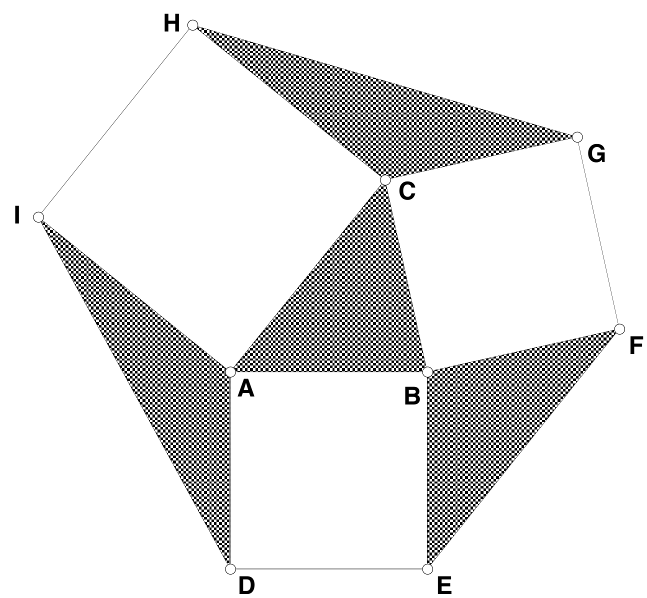

Figure 1

4. (In this problem we continue to prove what you discovered in Problem 2 of Assignment 7.) See Figure 2. Given: G is the centroid of $\triangle ABC$, M, N and P are the midpoints of AB, AC and BC respectively, and U, V, W, X, Y and Z are the centroids of the six "little triangles." To prove: the lines UV, XW and GB

are concurrent. (Hint: Use a proof similar to that of Theorem 29. You will find it helpful to consider the dashed lines in the picture. You may use Problem 6 of Assignment 8.)

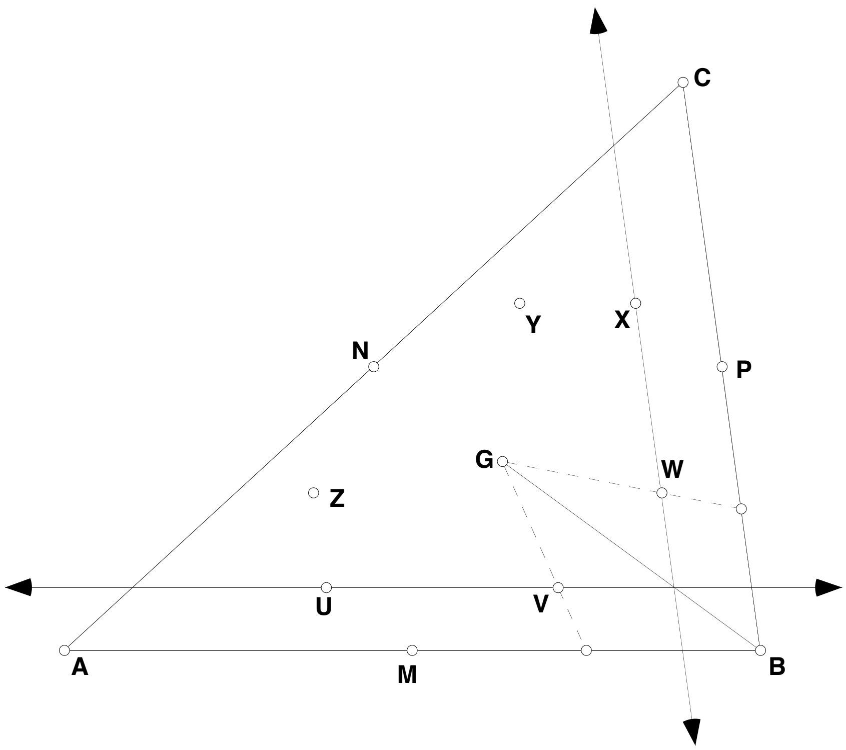

Figure 2

5. (See Figure 3). Given: R is the midpoint of BC. Prove that PQ is parallel to BC. (Hint: Use problem 3 from homework #7.)

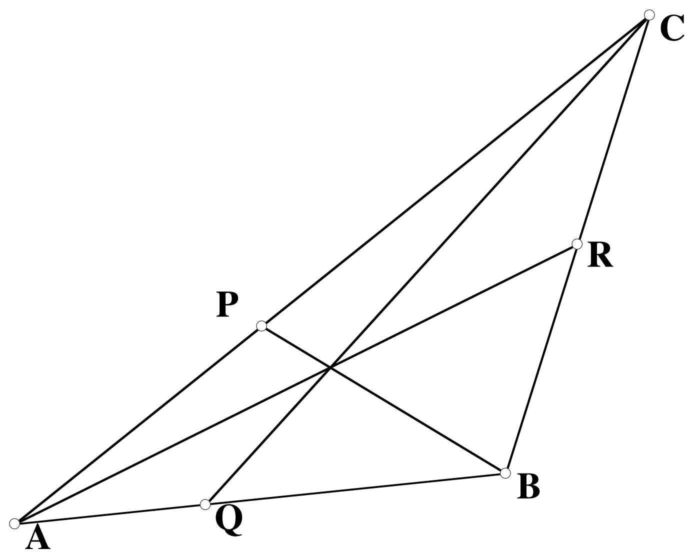

Figure 3

6. (See Figure 4.) This is a problem about three-dimensional geometry. Given: A, B and C are in the indicated plane, RBS is a straight line, $RB = SB$, $AB \perp RS$, and $\angle CAR = \angle CAS$. To prove: $\angle ACR = \angle ACS$ and $BC \perp RS$.

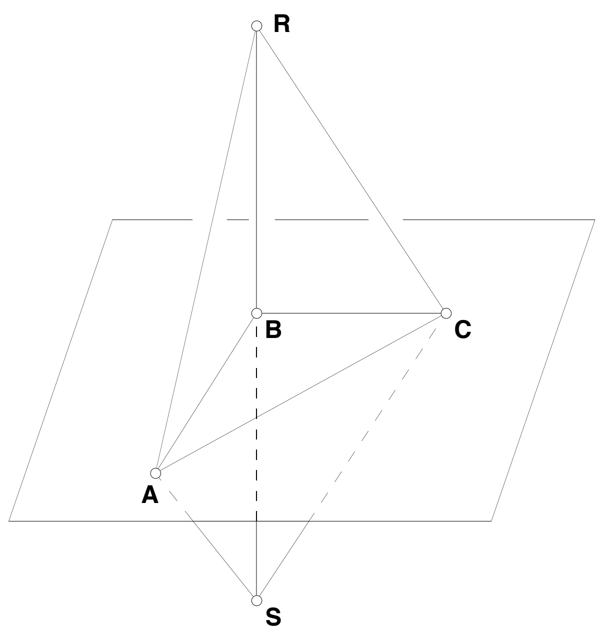

Figure 4

7. (15 points.) (In this problem we begin to prove the fact that you discovered in Problem 1 of Assignment 7.) See Figure 5. Given: M, N and P are the midpoints of AB, AC and BC respectively, MD is parallel to AP, and $MD = AP$.

   (a) Prove that the area of $\triangle CMD$ is twice the area of $\triangle CME$.

   (b) Prove that the area of $\triangle ABC$ is twice the area of $\triangle CMB$.

   (c) Prove that the area of $\triangle CME$ is 3/4 of the area of $\triangle CMB$.

   Hint: You may use what you proved about this picture in Problem 3 of Assignment 8. Also, use Theorem 14 as one ingredient.

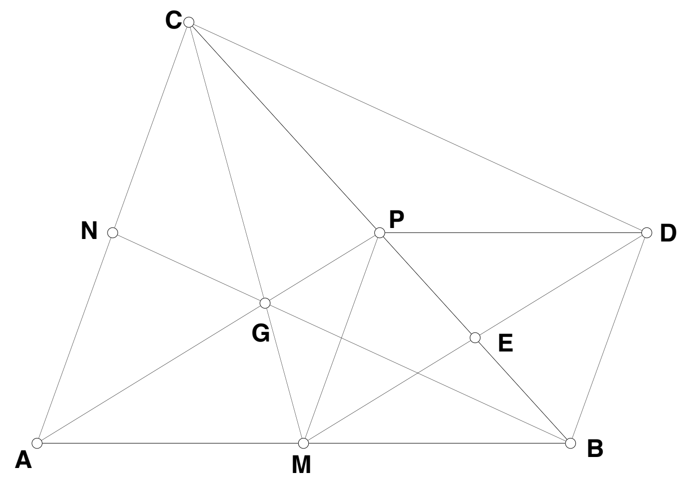

Figure 5

8. (See Figure 6) Given that $AB = BC$, $CD = DE$, and AE is parallel to BD, prove that $AE = 2BD$.

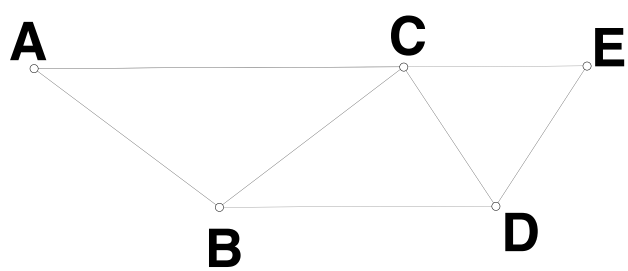

Figure 6

9. (See Figure 7.) Given: Angles ABC and AHG are right angles, and $ABDE$ and $ACFG$ are squares. To prove: $BC = 2HI$.

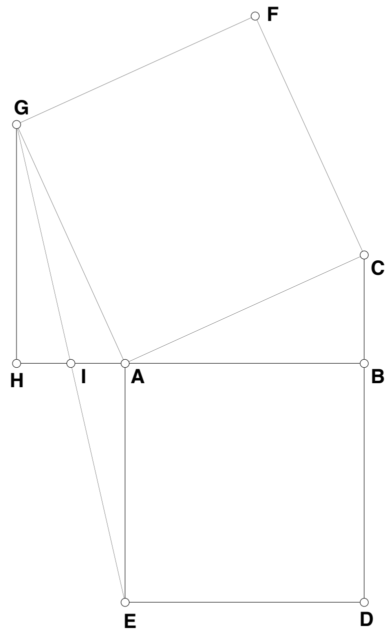

Figure 7

10. See Figure 8. Given $ABCD$ a quadrilateral, AE, BE, CG, and DG the angle bisectors of $\angle DAB, \angle ABC, \angle BCD$, and $\angle CDA$, respectively, and F and H the intersections of DG and AE, and CG and BE, respectively. Let P be the intersection of $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Show the lines AB, CD and FH are concurrent. (Hint: Use the result of problem 5 of assignment 6 as one ingredient of your proof).

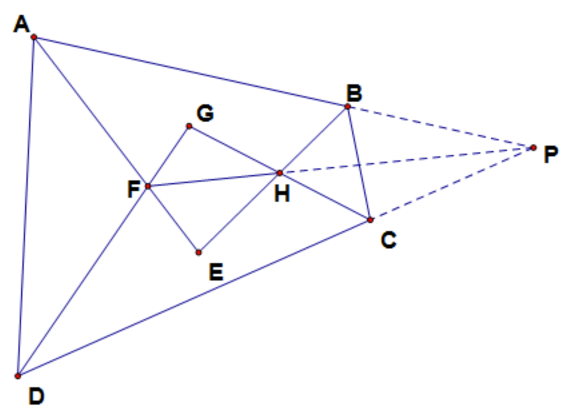

Figure 8