Exam III MA 174 4/21/00 D. Gottlieb

1a. Suppose it is 2 miles from town A to town B along a certain road. If you drive from A to B without stopping or turning around, what is the difference on your odometer between the mileage at town A and the mileage at town B?

b. The above simple question illustrates what equation expressing length as a line integral of a curve in terms of a parameter t?

2. Write down the vector field $\vec{F}$ for the vector field such that $\vec{F}(P)$ points towards the origin and is of unit length for any point P in $\mathbb{R}^3$. Use the $\vec{i}, \vec{j}, \vec{k}$ vectors.

3. Find a potential function for the vector field $\vec{F}(x, y, z) = z\vec{k}$.

4. Find the circulation around the ellipse $x^2 + \dfrac{z^2}{2} = 3$ of the vector field $\vec{F}$ in Problem 3.

5. Suppose an umbrella's shadow is the unit circle $x^2 + y^2 = 1$ when the sun is directly overhead on the z-axis. The umbrella can be described by the equation $f(x, y, z) = 0$ for some f whose gradient is given by $x\vec{i} + y^2\vec{j} + z^3\vec{k}$. Set up an integral giving the surface area of the umbrella. (You may leave z in your answer, since the information given does not completely describe z as a function of x and y.)

6. State Stokes's theorem with all its hypotheses.

7. Find the curl of the vector field in Problem 3.

8. Is a sphere ($x^2 + y^2 + z^2 = r^2$) simply connected?

9. Let

$$\vec{F} = \frac{-y}{x^2 + y^2}\ \vec{i}\ +\ \frac{x}{x^2 + y^2}\ \vec{j}\ +\ z\ \vec{k}$$

Now the curl of $\vec{F}$ is zero, and the circulation of $\vec{F}$ is not zero around the unit circle in the xy plane. Why can't you apply Stokes's Theorem to calculate the circulation on the unit circle? (If you are unsure of the reason, then show the curl of $\vec{F}$ is in fact zero and calculate the circulation of $\vec{F}$ around the unit circle in the xy plane.)

10. You want to use Stokes's theorem to calculate the circulation on the boundary of the surface S shown below. Can you choose a unit normal $\vec{n}$ to use in the surface integral of Stokes's theorem integrated over S? If so, what is it?