Functions Done Right

A *map* $f: X \to Y$ is a rule f which assigns to every object x in the *source* X an object $f(x)$ in the *target* Y.

Definition 2: Two sets X and Y are *equal*, denoted $X = Y$, $\Leftrightarrow$ every object of X is an object in Y and every object in Y is an object in X.

Definition 3: Two maps are equal $\Leftrightarrow$ Their sources are equal, their targets are equal, and their rules do the same thing.

In more symbolic notation this definition reads as follows: Let $f: X \to Y$ and $g: X' \to Y'$ be two maps . Then $f = g \Leftrightarrow X = X'$, and $Y = Y'$ and $f(a) = g(a)$ for all a in the source.

Definition 4. Suppose we have two maps, $f: X \to Y$ and $g: Y \to Z$ (note the target of f is the source of g). Then the map $h: X \to Z$ is the *composition* of f and $g \Leftrightarrow h(x) = g(f(x))$ for every x in the source X.

We say that $f(x)$ is the *image* of x, and that x is a *preimage* of $f(x)$. The set of all the preimages of y we will call the *fiber* of y.

The subset of the target Y which consists of elements y assigned to some x is called the *image* of f. Thus we can say that the *image* of f is the subset of objects of the target with non–empty fibers.

If A is a subset of the source X, then the *image* of A in the target Y consists of the set of images of all the objects in A. The image of A is denoted by $f(A)$. Similarly, the *preimage of a subset* B of the target Y is the set of all objects in the source X whose images are objects in B. The preimage of B is denoted by $f^{-1}(B)$.

If the *image* of f is the entire *target* of f, then we say that f is *onto.* Thus f is onto if and only if no fiber is empty. If the fibers of f have at most one object in it, then we say f is *one–to–one.* If every fiber of f has exactly one object in it, then we say that f is *one–to–one onto* or equivalently that f is *bijective.*