Gottlieb

Rules of Algebra

Addition

Let $a,\ b,\ c$ be any real numbers

1) $a+b=b+a$
2) $a+(b+c)=(a+b)+c$
3) $a+(-1)a=0$ (also written $a-a=0$)
4) $a+0=a$

Multiplication

5) $ab=ba$
6) $a(bc)=(ab)c$
7) $aa^{-1}=1$ (also written $\dfrac{a}{a}=1$) [if $a\neq 0$]
8) $1\cdot a=a,\ 0\cdot a=0,\ (-a)(-b)=ab$
9) $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}=(ad+bc)(bd)^{-1}$
10) $a(b+c)=ab+ac$

Exponents

11) $a^n=a\cdot a\ldots a$ (n–times) where n is a positive integer
12) $a^0=1$
13) $a^{-n}\!:=\frac{1}{a^n}$ $[a\neq 0]$
14) $a^{n/m}\!:=\sqrt[m/n]{a}$ (i.e. $(a^{n/m})^m=a^n$) [$a\geq 0$ if m is even]
15) $a^{r+s}=a^ra^s$ [for r,s any numbers and $a>0$]
16) $(a^r)^s=a^{rs}$ [for r,s any numbers and $a>0$]
17) $(ab)^r=a^rb^r$ [for r any number and $a>0,\ b>0$]
18) $1^r=1$
19) $0^r=0$ [if $r>0$]

Note that

20) $a-b\!:=a+(-1)b$
21) $\dfrac{a}{b}\!:=ab^{-1}$
22) $a^{m/n}\!:=\sqrt[m/n]{a}$ where $(a^{m/n})^n=a^m$

So $\sqrt{\ }$, subtraction, and $\dfrac{a}{a}$ are not necessary.