Review of "Poincaré and the Three Body Problem" by June Barrow-Green
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In a work of impressive scholarship, the author takes us through the history of the n body problem from Newton to the present. The center of her story is the prize competition in honor of the 60th birthday of King Oscar II of Sweden in 1889. With royal patronage, with the most prestigious mathematicians as judges, and with the most momentous mathematical problem of Civilization as a topic, it had captured the attention of the mathematical world. And the winner was ... Henri Poincaré ... with a manuscript that had a major error!

The paper was due to be published on the King's birthday a few weeks hence, when Poincaré himself discovered the false result. The difficulty of his position was enormous. An error in a paper so highly honored not only would be a great personal embarrassment, but it would damage the reputations of the judges and the organizers of the competition as well as ruin the Kings birthday.

Poincaré wrote the letter (surely the most difficult one a mathematician ever had to write), stopped the presses, paid for the printing costs (which exceeded the prize money by 1000 Kroner), and worked feverishly on a new manuscript, which was printed a year later. The letter and the first suppressed manuscript remained hidden in the Mittag-Leffler Institut archives and were only recently rediscovered.

The author analyzes the suppressed flawed manuscript along with the published corrected copy. What underlay Poincaré's error "is arguably the first description of chaotic motion within a dynamical system." The author goes into mathematical detail in tracing the influence of this manuscript, and of later ones by Poincaré on the subjects of Differential Equations, Dynamical Systems, and Celestial Mechanics. This mathematical detail should be interesting for the practitioners of those disciplines, for she has a clear way of describing research. For the rest of us, she tells about the controversy between the dynamical astronomers and Poincaré, the final solution to the three body problem, the mathematical personalities and politics of the competition, and much else. Her scholarship gives a firm historical base for reflections about what a mathematician really is.

For example, Arthur Jaffe and Frank Quinn's controversial article [1] discusses the issue of published results with inadequate proofs. Among the cautionary tales mentioned is Poincaré's discovery of homology. "Poincaré claimed too much, proved too little, and his *reckless* methods could not be imitated. The result was a dead area which had to be sorted out before it could take off." The context of these remarks imputes a kind of dishonesty to Poincaré, and claims it retarded the subject for years. However, in view of Poincaré's letter, we can ask the author of those lines whether it seems to him now that Poincaré was dishonest, and we can inquire if he himself would write such a letter if he were in Poincaré's position. As far as the "damage" done by Poincaré, I point out that some of the greatest mathematicians of the time took fifty years before they finally got homology right, and in the process they fundamentally changed the way we view almost all of Mathematics.

Practitioners of mathematics follow two historical traditions. One stems from the dawn of Civilization, the other arose in the time of the Greeks. In the older tradition, Mathematics is the handmaiden of the Arts, Science, and Industry. In the Greek tradition, Mathematics is the Queen of Knowledge, the only real way to Understand Nature.

By "Understand Nature", I mean understand it in the way that a Mathematician understands Mathematics: Clearly, distinctly, without ambiguity. To paraphrase Galileo: Once one tastes this kind of Knowledge, he can never be satisfied with a less perfect kind. Only a Mathematician can taste this kind of Knowledge. (Here I mean Mathematician in an inclusive sense, as opposed to merely a member of a particular profession.) This kind of Knowledge makes Mathematics the Queen of the Sciences, and she will reign forever.

But Mathematics is also the Handmaiden of the Sciences. It is a collection of tools to solve problems, to obtain answers, to describe and to measure and to name. You use it to build a bridge, to survey the land, or to navigate the sea. It perfected the masterpieces of our great painters and cast the horoscopes of our superstitious ancestors. This tradition is much older than the Greek tradition of Mathematics as a pure kind of knowledge, and for most well educated people it forms their view of what Mathematics is. Those who hold to this tradition may practice their mathematics with skill, but the mathematics is secondary to other considerations. I will call these people Practitioners.

Now among the fascinating things in this story is that a Practitioner named Hugo Gyldén, upon obtaining information about Poincaré's original prize winning paper, claimed he in fact had already published all of Poincaré's results. This led to a long controversy between those Practitioners called Dynamical Astronomers, and the Mathematicians. The problem was that the Mathematicians and the Astronomers had different ideas about what convergence of a series means:

"To illustrate how mathematicians and astronomers differed over this question, Poincaré compared the possible interpretations of the following two series

$$\Sigma(1000)^n/n! \quad \text{and} \quad \Sigma n!/(1000)^n$$

.

He argued that a mathematician would consider the first convergent and the second divergent, while an astronomer would label them the other way round."

This must be the best of the math versus physics jokes, because it is true! Yet Poincaré did not convey any criticism. He merely wanted to explain the difference to eliminate misunderstandings. He understood that truncating a divergent series whose initial terms decrease fast could produce numbers that are useful in practical problems, but he pointed out that such methods should not be used to prove theoretical results. And he observed that for practical computations it really does not matter whether or not the series converges: What is important is to have some idea of the upper bound of the errors involved.

Poincaré always said he learned a great deal from these Practitioners, including Gyldén. Most of us would react in the spirit of Hermite, who " was not impressed by Gyldén's grasp of analysis, describing Gyldén as a ghost from a bygone age, who had been left behind as the world of analysis transformed about him." It turns out, though, that Poincaré had the right point of view, because in 1909, the Finnish astronomer Karl Sundman completely solved the three-body problem! Given an initial position, he could produce a convergent series giving the positions of the bodies for all times.

Wait a minute, I didn't know that the Three Body Problem was solved. I'll bet you didn't either. " Sundman's work seems to have been almost forgotten. Why did such an important and long awaited work almost fade into obscurity?" Think about it!

The n body problem can be thought of as the most fateful problem in all of Mathematics. One might say that the mathematician Galileo Galilei "solved" the one body problem by assuming as an axiom that a body moves with uniform motion in a straight line. Reasoning with other axioms, he showed that a projectile follows a parabolic path. To objections that no one has seen a body move forever in a straight line, he would say: Let us derive the mathematics, and compare the results to those we actually observe. If their differences can be explained by other effects, then the axiom is reasonable. If there is a disparity, then the axiom should be discarded. Thus Galileo followed in the tradition of Archimedes and used Mathematics to Understand Nature.

The two body problem was essentially solved by Newton in 1687. Newton laid down his three Laws, the first two adapted from Galileo, plus the fourth Law of Gravity. With these axioms mathematicians could understand the workings of the solar system, and they strove to develop methods of calculating. This stupendous achievement, Newtonian Mechanics, led to an entire reorientation of Western Culture. The following century, called the Age of Reason, found Mathematical ideas applied in every field as people tried to emulate Newton's clarity.

How could the solution of the three body problem fade into obscurity? Well, the first reason is that Sundman's series does not converge according to the Practitioners. The second reason is that it is an algorithm conveying little insight: although it is precise, it does not add much to Understanding Nature. The third reason is that the Physicist Albert Einstein, noting some slight inadequacy in Galileo's solution of the one body problem, propounded a new solution, and thereby became a Mathematician.

I thoroughly enjoyed June Barrow-Green's book. I have written things here I would not have dreamt of saying before reading it. For me, the center of the work is Poincaré's letter; for now we can show the Practitioners, and the World, just what we Mathematicians are.

REFERENCE

[1] Jaffe, Arthur; Quinn, Frank "Theoretical mathematics": toward a cultural synthesis of mathematics and theoretical physics. Bull. Amer. Math. Soc. (N.S.) 29 (1993), no. 1, 1–13.