

## Financial Mathematics Exam—April 2018

The Financial Mathematics exam is a three-hour exam that consists of 35 multiple-choice questions and is administered as a computer-based test. For additional details, please refer to [Exam Rules](#)

The goal of the syllabus for this examination is to provide an understanding of the fundamental concepts of financial mathematics, and how those concepts are applied in calculating present and accumulated values for various streams of cash flows as a basis for future use in: reserving, valuation, pricing, asset/liability management, investment income, capital budgeting, and valuing contingent cash flows.

The Financial Mathematics Exam assumes a basic knowledge of calculus and an introductory knowledge of probability.

The following learning objectives are presented with the understanding that candidates are allowed to use specified calculators on the exam. The education and examination of candidates reflects that fact. In particular, such calculators eliminate the need for candidates to learn and be examined on certain mathematical methods of approximation.

Please check the [Updates](#) section on this exam's home page for any changes to the exam or syllabus.

Each multiple-choice problem includes five answer choices identified by the letters A, B, C, D, and E, only one of which is correct. Candidates must indicate responses to each question on the computer. Candidates will be given three hours to complete the exam.

As part of the computer-based testing process, a few pilot questions will be randomly placed in the exam (paper and pencil and computer-based forms). These pilot questions are included to judge their effectiveness for future exams, but they will NOT be used in the scoring of this exam. All other questions will be considered in the scoring. All unanswered questions are scored incorrect. Therefore, candidates should answer every question on the exam. There is no set requirement for the distribution of correct answers for the multiple-choice preliminary examinations. It is possible that a particular answer choice could appear many times on an examination or not at all. Candidates are advised to answer each question to the best of their ability, independently from how they have answered other questions on the examination.

Since the CBT exam will be offered over a period of a few days, each candidate will receive a test form composed of questions selected from a pool of questions. Statistical scaling methods are used to ensure within reasonable and practical limits that, during the same testing period of a few days, all forms of the test are comparable in content and passing criteria. The methodology that has been adopted is used by many credentialing programs that give multiple forms of an exam.

The ranges of weights shown in the Learning Objectives below are intended to apply to the large majority of exams administered. On occasion, the weights of topics on an individual exam may fall outside the published range. Candidates should also recognize that some questions may cover multiple learning objectives.

## LEARNING OBJECTIVES

- A. Time Value of Money (10-15%)
1. The candidate will be able to define and recognize the definitions of the following terms: interest rate (rate of interest), simple interest, compound interest, accumulation function, future value, current value, present value, net present value, discount factor, discount rate (rate of discount), convertible m-thly, nominal rate, effective rate, inflation and real rate of interest, force of interest, equation of value.
  2. The candidate will be able to:
    - a. Given any three of interest rate, period of time, present value, current value, and future value, calculate the remaining item using simple or compound interest. Solve time value of money equations involving variable force of interest.
    - b. Given any one of the effective interest rate, the nominal interest rate convertible m-thly, the effective discount rate, the nominal discount rate convertible m-thly, or the force of interest, calculate any of the other items.
    - c. Write the equation of value given a set of cash flows and an interest rate.
- B. Annuities/cash flows with payments that are not contingent (15-20%)
1. The candidate will be able to define and recognize the definitions of the following terms:  
annuity-immediate, annuity due, perpetuity, payable m-thly or payable continuously, level payment annuity, arithmetic increasing/decreasing annuity, geometric increasing/decreasing annuity, term of annuity.
  2. For each of the following types of annuity/cash flows, given sufficient information of immediate or due, present value, future value, current value, interest rate, payment amount, and term of annuity, the candidate will be able to calculate any remaining item.
    - a. Level annuity, finite term
    - b. Level perpetuity
    - c. Non-level annuities/cash flows
      - i) Arithmetic progression, finite term
      - ii) Arithmetic progression, perpetuity
      - iii) Geometric progression, finite term
      - iv) Geometric progression, perpetuity
      - v) Other non-level annuities/cash flows
- C. Loans (15-20%)
1. The candidate will be able to define and recognize the definitions of the following terms: principal, interest, term of loan, outstanding balance, final payment (drop payment, balloon payment), amortization, sinking fund.
  2. The candidate will be able to:
    - a. Given any four of term of loan, interest rate, payment amount, payment period, principal, calculate the remaining item.
    - b. Calculate the outstanding balance at any point in time.
    - c. Calculate the amount of interest and principal repayment in a given payment.
    - d. Given the quantities, except one, in a sinking fund arrangement calculate the missing quantity.
    - e. Perform similar calculations to a-d when refinancing is involved.

- D. Bonds (15-20%)
1. The candidate will be able to define and recognize the definitions of the following terms: price, book value, amortization of premium, accumulation of discount, redemption value, par value/face value, yield rate, coupon, coupon rate, term of bond, callable/non-callable.
  2. Given sufficient partial information about the items listed below, the candidate will be able to calculate the any of the remaining items.
    - a. Price, book value, amortization of premium, accumulation of discount
    - b. Redemption value, face value
    - c. Yield rate
    - d. Coupon, Coupon rate
    - e. Term of bond, point in time that a bond has a given book value, amortization of premium, or accumulation of discount
- E. General Cash Flows and Portfolios (10-15%)
1. The candidate will be able to define and recognize the definitions of the following terms: yield rate/rate of return, dollar-weighted rate of return, time-weighted rate of return, current value, duration (Macaulay and modified), convexity (Macaulay and modified), portfolio, spot rate, forward rate, yield curve, stock price, stock dividend.
  2. The candidate will be able to:
    - a. Calculate the dollar-weighted and time-weighted rate of return.
    - b. Calculate the duration and convexity of a set of cash flows.
    - c. Calculate either Macaulay or modified duration given the other.
    - d. Use duration to approximate the change in present value due to a change in interest rate.
      - i. Using 1<sup>st</sup>-order linear approximation based on modified duration.
      - ii. Using 1<sup>st</sup>-order approximation based on Macaulay duration.
    - e. Calculate the price of a stock using the dividend discount model.
- F. Immunization (10-15%)
1. The candidate will be able to define and recognize the definitions of the following terms: cash flow matching, immunization (including full immunization), Redington immunization.
  2. The candidate will be able to:
    - a. Construct an investment portfolio to fully immunize a set of liability cash flows.
    - b. Construct an investment portfolio to match present value and duration of a set of liability cash flows.
    - c. Construct an investment portfolio to exactly match a set of liability cash flows.
- G. Interest Rate Swaps (0-10%)
1. The candidate will be able to define and recognize the definitions of the following terms: swap rate, swap term or swap tenor, notional amount, market value of a swap, settlement dates, settlement period, counterparties, deferred swap, amortizing swap, accreting swap, interest rate swap net payments.
  2. The candidate will be able to:
    - a. Calculate the swap rate in an interest rate swap, deferred or otherwise, and with either constant or varying notional amount.
    - b. Calculate the market value of an interest rate swap, deferred or otherwise, and with either constant or varying notional amount.

H. Determinants of Interest Rates (0-10%)

1. The candidate will be able to define and recognize the components of interest rates including: real risk-free rate, inflation rate, default risk premium, liquidity premium, and maturity risk premium.
2. The candidate will be able to identify the real interest and the nominal interest rate in the context of loans with and without inflation protection and calculate the effect of changes in inflation on loans with inflation protection.
3. The candidate will be able to explain how the components of interest rates apply in various contexts, such as commercial loans, mortgages, credit cards, bonds, government securities.
4. The candidate will be able to explain the roles of the Federal Reserve and the FOMC in carrying out fiscal policy and monetary policy and the tools used by the Federal Reserve and the FOMC including targeting the Federal Funds rate, setting reserve requirements, and setting the discount rate.
5. The candidate will be able to explain the theories of why interest rates differ by term, including liquidity preference (opportunity cost), expectations, preferred habitat, and market segmentation.
6. The candidate will be able to explain how interest rates differ from one country to another (e.g., U.S. vs. Canada).

**Text References**

Knowledge and understanding of the financial mathematics concepts are significantly enhanced through working out problems based on those concepts. Thus, in preparing for the Financial Mathematics exam, whichever of the source textbooks candidates choose to use, candidates are encouraged to work out the textbook exercises related to the listed readings.

**Suggested Textbooks for Learning Objectives in Section I, Interest Theory**

There is not a single textbook required for the learning objectives in Section I. The texts listed below are representative of the textbooks available to cover the material on which the candidate may be tested. Not all topics may be covered at the same level in each text. Listed sections may include introductory material, summary material, and problems that are not part of the learning objectives. The candidate may wish to use one or more texts in his/her preparation for the examination.

Broverman, S.A., *Mathematics of Investment and Credit (Sixth Edition)*, 2015, ACTEX

Publications, ISBN 978-1-62542-485-3:

Chapter 1 (excluding 1.2.1, 1.7, and 1.8)

Chapter 2 (excluding 2.4.2 and 2.4.3)

Chapter 3 (excluding 3.2.1, 3.2.2 and 3.4)

Chapter 4 (excluding 4.3.2)

Chapter 5 (excluding 5.1.4, the investment year method portion of 5.3.1, 5.3.2 and 5.3.3)

Chapter 6 (excluding 6.2 and 6.4)

Chapter 7 (excluding 7.1.6 and 7.3)

In addition, candidates are not responsible for formula (7.6) and its uses. Thus, excluded is the part of 7.1.2 from the paragraph starting "Although" on page 369 to the end of that section, the part of 7.1.3 from the paragraph starting "It" on page 372 to the end of that section, and the last sentence of 7.1.5.

Chapter 9 (9.1 only)

At various places in the sections of this text that are listed above there are statements indicating that more information is available in sections that are not listed above. Candidates are not responsible for this additional information.

Daniel, J.W., and Vaaler, L.J.F., *Mathematical Interest Theory* (Second Edition), 2009, The Mathematical Association of America, ISBN: 978-0883857540 : **[Candidates may also use the First Edition of *Mathematical Interest Theory* (Publisher: Prentice Hall, ISBN: 0-13-147285-2). The same chapter references apply.]**

Chapter 1 (excluding 1.13 and 1.14)

Chapter 2

Chapter 3 (excluding 3.10, 3.12, and the investment year method portion of 3.13)

Chapter 4

Chapter 5

Chapter 6 (excluding 6.10)

Chapter 7 (excluding 7.2, 7.3, and 7.4)

Chapter 8 (8.3 only)

Chapter 9 (excluding 9.5)

Kellison, S.G., *The Theory of Interest* (Third Edition), 2009, Irwin/McGraw-Hill, ISBN: 125921544X or 978-1259215445:

Chapter 1

Chapter 2

Chapter 3 (excluding 3.9)

Chapter 4

Chapter 5 (excluding 5.7 and 5.8)

Chapter 6 (excluding 6.8, 6.9, and 6.11)

Chapter 7 (excluding the investment year method portion of 7.7, 7.8, 7.9, and 7.10)

Chapter 10 (excluding 10.6 and 10.7)

Chapter 11 (excluding 11.4 and 11.9)

Francis, J. and Ruckman, C., *Interest Theory – Financial Mathematics and Deterministic Valuation*; (First Edition), 2016, ActuarialBrew, ISBN 978-0998160405

Chapter 1

Chapter 2

Chapter 3

Chapter 4

Chapter 5

Chapter 6

Chapter 7

Chapter 8

Chapter 9

Chapter 10

Chapter 11

Chapter 12 (excluding 12.04)

Chapter 13 (excluding 13.04)

Chapter 14

Chapter 15

Chapter 16

Chan, Wai-Sum, and Tse, Yiu-Kuen, *Financial Mathematics for Actuaries*, Updated Edition, 2013, McGraw-Hill Education (Asia), ISBN: 978-1259011481 **[Candidates may also use the 2011 Edition of *Financial Mathematics for Actuaries* with [Supplementary Notes for Financial Mathematics for Actuaries](#) for Chapter 8. The same chapter references apply.]**

Chapter 1

Chapter 2 (excluding 2.4)
Chapter 3
Chapter 4 (excluding 4.5, the investment year method portion of 4.6, 4.7, and 4.8)
Chapter 5
Chapter 6
Chapter 7
Chapter 8 (excluding 8.6, 8.7, and 8.8)

#### ADDITIONAL REFERENCES

There are three study notes that are required reading for this examination. They can be downloaded from this document by clicking on the links.

- For Learning Objective E.2.d.ii – A study note on using Macaulay duration to approximate the effect of interest rate changes on present values. It is [FM-24-17 Using Duration and Convexity to Approximate Change in Present Value](#). Sections 1-4 are required reading for this examination.
- For Learning Objective G – A study note on interest rate swaps. It is [FM-25-17 Interest Rate Swaps](#). The entire note is required reading.
- For Learning Objective H – A study note on the determinants of interest rates. It is [FM-26-17 Determinants of Interest Rates](#). The entire note is required reading.

#### OTHER RESOURCES:

[Notation and terminology used for Exam FM](#)

[All released exam papers](#), since 2000, can be found here.

Sample [Questions](#) and [Solutions](#)

[Review of Calculator Functions for the Texas Instruments BA-35](#)

[Review of Calculator Functions for the Texas Instruments BA II Plus](#)

Although several different calculators are allowed for this exam, the BAII Plus is recommended due to its ability to solve for interest rates.

[Online Sample Exam FM](#)

The Society of Actuaries (SOA) is interested in supporting candidates as they prepare for the preliminary exams. To that end the SOA has launched an online sample exam for Exam FM (Financial Mathematics). Available at no cost the sample exam selects questions and solutions in an online exam experience that resembles the computer-based testing employed for most of the SOA's preliminary exams. Questions have been coded to meet the Exam FM learning objectives and ensure candidates receive a balanced yet randomized set of questions each time they repeat the sample exam. The current set of questions is drawn from the existing set of sample questions.