MATH 373
Test 4
Fall 2017

December 12, 2017

1. A three year bond has an annual coupon of 40 and a maturity value of 1100.

   Calculate the Modified Convexity for this bond at an annual effective interest rate of 6.5%.

2. Stanley Insurance Company has agreed to pay Jacqueline 400,000 at the end of each year for three years. Stanley wants to use the following three bonds to exactly match the payments to Jacqueline.

    a. Bond A is a one year bond with a maturity value of 1000, annual coupons of 70, and a price of 1005.

    b. Bond B is a two year bond with annual coupons of 200 and a maturity value of 1000. It sells to yield an annual effective interest rate of 8%.

    c. Bond C is a three year bond with annual coupons of 270. The maturity value and the price of this bond is 3000.

    Calculate the cost to buy the bonds to exactly match the payments. Assume that we can purchase partial bonds.

3. A 30 year bond matures for 10,000 and has semi-annual coupons of 300.

   Calculate the Modified duration at an annual effective interest rate of 10.25%.

4. A three year bond with annual coupons of 300 matures for 4000.

   Calculate the Macaulay Convexity of this bond at an annual effective rate of 5%.

5. The Moses Life Insurance Company owns the following two bonds:

	Price	Macaulay Duration	Macaulay Convexity
Bond A	60,000	A	90
Bond B	40,000	12	125

Using an interest rate of 8%, the Modified Convexity of this bond portfolio is 98.654.

Determine A.

6. Billie has promised to make a payment of 1,000,000 to Bokun at the end of 7.5 years. Billie wants to immunize this payment using Reddington Immunization and the following two zero coupon bonds:

    a. Bond 1 is a zero coupon bond with a maturity value of 10,000 at the end of 4 years.

    b. Bond 2 is a zero coupon bond with a maturity value of 15,000 at the end of 8 years.

    Assuming an interest rate of 8%, determine the number of Bond 1 which Billie should purchase. Assume that we can purchase partial bonds.

7. A perpetuity due pays 100 at the start of each year.

   Calculate the Macaulay Duration at an interest rate of 6%.

8. Graham owns a bond with a price of 130,000 at an annual effective yield rate of 7%. The bond has a Modified Duration of 14.0187 and a Modified Convexity of 100 at an annual effective interest rate of 7%.

   Graham estimates that price of this bond at an interest rate of i using the first order Macaulay approximation to be 117,886.61.

   Determine i.

9. You are given the following spot interest rates:

Time (t)	r_t
0.5	2.1%
1.0	2.3%
1.5	2.6%
2.0	3.0%
2.5	3.3%
3.0	3.7%
3.5	4.2%
4.0	4.8%
4.5	5.5%
5.0	6.0%

Megan purchases a two year par value bond with semi-annual coupons at a rate of 8% convertible semi-annually. The par value of the bond is 10,000.

Calculate the price of the bond.

10. The following three bonds are priced using the same spot interest rates:

    a. Bond A is a one year bond with a maturity value of 1000, annual coupons of 70, and a price of 1005.

    b. Bond B is a two year bond with annual coupons of 200 and a maturity value of 1000. It sells to yield an annual effective interest rate of 8%.

    c. Bond C is a three year bond with annual coupons of 270. The maturity value and the price of the bond are 3000.

    Kristin has a three year annuity immediate with annual payments of 1000. Using the same spot interest rates, determine the accumulated value of Kristin's annuity.

11. You are given:

a. $f_{[0,1]} = 0.05$

b. $f_{[1,2]} = 0.06$

c. $f_{[2,3]} = 0.07$

d. $f_{[3,4]} = 0.08$

Calculate the price of a zero coupon bond that matures for 100,000 at the end of four years.

You are given the following spot interest rates and information for Questions 12-15:

Time t	Spot Rate r_t
1	0.053
2	0.057
3	0.063
4	0.071
5	0.080

The F&G Corporation borrows 700,000 from Abbott Bank. The terms of the loan are such that F&G will pay a variable interest rate on the loan each year and will repay the 700,000 at the end of four years. The variable interest rate will be the one year spot rate at the beginning of each settlement period.

F&G is uncomfortable with the risk of having a variable loan interest rate. Therefore, F&G purchases a four year interest rate swap from M&J Investment Bank. At the end of each year, F&G will pay a fixed rate to M&J Investment Bank. At the end of each year, M&J will pay the variable rate to F&G. The amount of the payment will be based on the amount of the loan of 700,000.

12. Answer the questions below. These four parts are worth one question.

    a. List the Payer under this agreement.

    b. List the Receiver under this agreement.

    c. State the Settlement Period under the agreement.

    d. List the notional amount in the first year under this agreement.

13. Calculate the swap rate under this agreement.

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14. Based on the spot interest rate curve at the time of the loan, what is the implied rate that F&G would pay to Abbott Bank in the third year of the loan?

15. Calculate the net swap payment at the end of the first year. State who receives the payment and who makes the payment.

16. You are given the following spot interest rates:

Time t	Spot Rate r_t
1	0.053
2	0.057
3	0.063
4	0.071
5	0.080

Thomas and Brett enter into a five year deferred interest rate swap. There will be no swap during the first three years. The settlement periods will be one year. Under the swap, Thomas agrees to pay a variable interest rate to Brett at the end of the fourth year and at the end of the fifth year. The variable interest rate is based on the one year spot interest rate at the start of each year.

In return, Brett agrees to pay a fixed rate to Thomas at the end of the fourth year and the fifth year.

The notional swap amount is 100,000 for both settlement periods.

Calculate the swap rate for this swap.

17. The current spot interest rate curve is as follows:

t	r_t		t	r_t
0.25	2.50%		1.75	3.40%
0.50	2.65%		2.00	3.48%
0.75	2.79%		2.25	3.80%
1.00	2.92%		2.50	4.10%
1.25	3.10%		2.75	4.35%
1.50	3.25%		3.00	4.50%

Cai has a two year loan for 500,000 which has a variable interest rate that resets at the beginning of each six month period. The interest rate will be the six month spot interest rate at the beginning of each six month period.

Cai enters into an interest rate swap where she is the payer. The characteristics of the swap mirror those of the loan.

Determine the six month swap rate that Cai will pay.

18. Sue and Gavin enter into a three year swap. Under the swap, Sue is the payer and Gavin is the receiver. The swap has annual settlement periods. The variable rate to be paid is the one year spot rate at the beginning of each year for the next three years.

The following is the spot interest rate curve:

Time t	**Spot Rate r_t**
1	0.060
2	0.055
3	0.050
4	0.048
5	0.046

The notional amount for the swap changes each year. The notional amount during the first year is 500,000. The notional amount during the second year is 300,000. The notional amount during the third year is 100,000.

Calculate the swap rate for this swap.