MATH 373
Test 1
Fall 2018
October 4, 2018

1. Walker borrows 25,000 at an annual effective interest rate of 8%. Walker will repay the loan with three payments over the next five years. The first payment will be $3P$ at the end of 2 years. The second payment will be P at the end of 3 years. The final payment will be $2P$ at the end of five years.

   Determine P .

Solution:

$$25,000 = 3P(1.08)^{-2} + P(1.08)^{-3} + 2P(1.08)^{-5}$$

$$P = \frac{25,000}{3(1.08)^{-2} + (1.08)^{-3} + 2(1.08)^{-5}} = 5288.75$$

Or

$$25,000(1.08)^{5} = 3P(1.08)^{3} + P(1.08)^{2} + 2P$$

$$P = \frac{25,000(1.08)^{5}}{3(1.08)^{3} + (1.08)^{2} + 2} = 5288.75$$

2. Yoon Industries invests X to build a factory. Yoon expects to receive the following cash flows over the next 4 years.

End of Year	Cash Flow
1	Negative 1 million
2	Positive 2 million
3	Positive 3 million
4	Positive 4 million

At the end of four years, the factory is obsolete and will not generate any further cash flows.

The internal rate of return on this investment is 10%.

Calculate the net present value at 15%.

Solution:

$$0 = X - 1(1.10)^{-1} + 2(1.10)^{-2} + 3(1.10)^{-3} + 4(1.10)^{-4}$$

$$X = 1(1.10)^{-1} - 2(1.10)^{-2} - 3(1.10)^{-3} - 4(1.10)^{-4} = -5.729799877 \text{ million}$$

$$NPV = -5.729799877 - 1(1.15)^{-1} + 2(1.15)^{-2} + 3(1.15)^{-3} + 4(1.15)^{-4}$$

$$= -0.82751608 \text{ million} = -827{,}516.08$$

3. Alex invests 10,000 in an account earning simple interest at a rate of 8%.

   Quinn invests S in an account earning 6% compounded continuously.

   At the end of 10 years, Alex and Quinn have the same amount.

   Yash invests S at an annual interest rate equivalent to a discount rate of 7%.

   Determine the amount that Yash will have at the end of 10 years.

   Solution:

   $$Alex: 10,000[1+0.08(10)] = 18,000$$

   $$Quinn: Se^{(0.06)(10)} = 18,000 ==> S = \frac{18,000}{e^{0.6}} = 9878.60945$$

   $$Yash: 9878.60945(1-0.07)^{-10} = 20,411.10$$

4. Taylen purchased a 90 day US Treasury Bill which will mature for 100,000 and has a quoted rate of 6%.

   Calculate the Price that Taylen paid to purchase this Treasury Bill.

   Solution:

$$\text{Quoted Rate} = 0.06 = \frac{360}{90}\frac{100,000 - \text{Price}}{\text{Maturity Value}}$$

$$0.015 = \frac{100,000 - \text{Price}}{100,000}$$

$$1500 = 100,000 - \text{Price}$$

$$\text{Price} = 100,000 - 1500 = 98,500$$

5. Kanishk loans Ginuli 10,000 which will be repaid with three annual payments of 4500. Kanishk takes each payment from Ginuli and reinvests the payment at an annual effective interest rate of $r\%$.

The annual return earned by Kanishk taking into account reinvestment was 14%.

Determine $r\%$.

Solution:

$$4500(1+r)^2 + 4500(1+r) + 4500 = 10,000(1.14)^3 = 14,815.44$$

$$4500(1+r)^2 + 4500(1+r) - 10,315.44 = 0$$

$$\text{Let } x = 1+r$$

$$4500x^2 + 4500x - 10,315.44 = 0$$

$$x = \frac{-4500 \pm \sqrt{4500^2 - 4(4500)(-10,315.44)}}{(2)(4500)} = 1.094465428$$

$$r = 0.094465$$

6. Connor has borrowed money from Brow Bank. The loan has an annual effective interest rate of 5.25%.

   Connor has agreed to repay the loan with 10 annual payments. The first payment is 100,000. The second payment is 90,000. Each payment is 10,000 less than the prior payment until a payment of 10,000 is paid at the end of the 10 year.

   Determine the outstanding loan balance on this loan right after the seventh payment.

   Solution:

   $$OLB_7 = \text{Present Value of Future Payments}$$

   $$=30{,}000(1.0525)^{-1} + 20{,}000(1.0525)^{-2} + 10{,}000(1.0525)^{-3}$$

   $$= 55{,}135.04$$

7. You are given:

   a. $v(t)=\dfrac{1}{\beta+0.01t+\alpha t^2}$

   b. $i_{11}=0.04$

Calculate δ_{10} .

Solution:

$$v(t)=\frac{1}{a(t)} ==> a(t)=\beta+0.01t+\alpha t^2$$

$$a(0)=1 ==> \beta+0.01(0)+\alpha(0^2)=1 ==> \beta=1$$

$$i_{11}=0.04 ==> \frac{a(11)-a(10)}{a(10)}=0.04 ==> \frac{1+0.01(11)+\alpha(11^2)-[1+0.01(10)+\alpha(10^2)]}{1+0.01(10)+\alpha(10^2)}=0.04$$

$$==> \frac{0.11+121\alpha-0.10-100\alpha}{1+0.1+100\alpha}=0.04 ==> 0.01+21\alpha=0.04+0.004+4\alpha$$

$$==> 17\alpha=0.034 ==> \alpha=0.002$$

$$a(t)=1+0.01t+0.002t^2$$

$$\delta_t=\frac{a'(t)}{a(t)}=\frac{0.01+0.004t}{1+0.01t+0.002t^2} ==> \delta_{10}=\frac{0.01+0.004(10)}{1+0.01(10)+0.002(10^2)}=0.03846$$

8. Huber Bank makes three year loans. Huber wants to receive an annual rate of 3.5% compounded continuously to defer consumption. Huber believes that the annual rate of inflation for the next three years will be 1.8% compounded continuously. Additionally, Huber charges an annual rate of 30 basis points compounded continuously for the risk that inflation could exceed expectations.

   Huber believes that 4% of the loans will default and the recovery rate on the defaulted loans will be 60%.

   Calculate the total annual interest rate compounded continuously that Huber will charge on these three year loans.

   Solution:

$$\text{Rate without Defaults} = 0.035 + 0.018 + 0.003 = 0.056$$

$$\text{Rate with Defaults} = 0.056 + \delta_s$$

$$e^{(0.056)(3)} = (1-0.04)e^{(0.056+\delta_s)(3)} + (0.04)(0.6)e^{(0.056+\delta_s)(3)} = 0.984e^{(0.056+\delta_s)(3)}$$

$$\frac{e^{(0.056)(3)}}{0.984} = e^{(0.056+\delta_s)(3)} \text{ ==> } (0.056+\delta_s)(3) = \ln\left[\frac{e^{(0.056)(3)}}{0.984}\right] = 0.184129382$$

$$\delta_s = \frac{0.184129382}{3} - 0.056 = 0.005376461$$

$$\text{Total Interest Rate} = 0.056 + 0.005376461 = 0.0613765$$

9. Adam is repaying a loan with monthly payments of 200 for 63 months. The interest rate on the loan is 9% compounded monthly.

   Determine the outstanding loan balance of this loan at the end of one year which is twelve months.

   Solution:

   $$i^{(12)} = 0.09$$

   $$\frac{i^{(12)}}{12} = 0.0075$$

   $$OLB_{12} = 200a_{\overline{63-12}|} = 200\left(\frac{1-(1.0075)^{-51}}{0.0075}\right) = 8449.92$$

10. On January 1, 2016, Natalie invested 100,000 in an account. On May 31, 2016, Natalie's account was worth 110,000 and she invested an additional 45,000. On August 1, 2016, Natalie's account was worth 150,000. At that time, Natalie withdrew 25,000 to pay her tuition. On December 31, 2017, Natalie's account was now worth 135,000 and Natalie decided to withdraw all her money.

    Calculate Natalie's annual time weighted return for this account.

    Solution:

$$1+j_1=\frac{110,000}{100,000}$$

$$1+j_2=\frac{150,000}{155,000}$$

$$1+j_3=\frac{135,000}{125,000}$$

$$1+j_{tw}=(1+j_1)(1+j_2)(1+j_3)=\left(\frac{110,000}{100,000}\right)\left(\frac{150,000}{155,000}\right)\left(\frac{135,000}{125,000}\right)=1.1496774$$

$$1+i_{TW}=(1+j_{TW})^{1/2}=1.0722301$$

$$i_{TW}=0.072230$$

11. The US Government issues four year inflation protected loans. The real interest rate compounded continuously during the four years is 2.5% annually. The US Government is considered a risk free borrower so there is no default charge.

    The annual inflation rate compounded continuously for the first two years of the loan is 4%. The annual inflation rate compounded continuously for the last two years of the loan is $i_a\%$.

    Spencer borrows 100,000 from the US Government and repays 122,556.26 at the end of four years.

    Determine $i_a\%$.

    Solution:

$$100,000e^{(0.025)(4)}e^{(0.04)(2)}e^{(i_a)(2)} = 122,556.26$$

$$e^{(i_a)(2)} = \frac{1.2255626}{e^{(0.025)(4)}e^{(0.04)(2)}} = 1.023675932$$

$$(i_a)(2) = \ln(1.023675932) ==> i_a = 1.17\%$$

12. Navneet is receiving 120 monthly payments of 125 from a Trust Fund.

    Calculate the current value of these payments right after the 78^{th} payment using an interest rate of 12% compounded monthly.

    Solution:

$$i^{(12)} = 0.12 ==> \frac{i^{(12)}}{12} = 0.01$$

First, calculate the present value of the payments:

$$PV = 125a_{\overline{120|}} = 125\left(\frac{1-(1.01)^{-120}}{0.01}\right) = 8712.565254$$

Then calculate the current value at time 78:

$$\text{PV}(1.01)^{78} = (8712.565254)(1.01)^{78} = 18,932.73$$

13. Krystian borrows 30,000 to buy a new car. The loan is a 54 month loan. There are no payments made at the end of the first six months. This deferral period is followed by 48 payments of Q with the first payment of Q made at the end of the seventh month and the last payment made at the end of the 54th month.

The interest rate on the loan is an annual effective interest rate of 6%.

Determine Q .

Solution:

$$i = 0.06 ==> \frac{i^{(12)}}{12} = (1.06)^{1/12} - 1 = 0.004867551$$

$$30,000 = Qv^6 a_{\overline{48}|} = Q(1.004867551)^{-6} \left(\frac{1-(1.004867551)^{-48}}{0.004867551} \right)$$

$$30,000 = Q(41.48626355)$$

$$Q = \frac{30,000}{41.48626355} = 723.13$$