MATH 373
Test 2
Fall 2018
November 1, 2018

1. A 20 year bond has a par value of 1000 and a maturity value of 1300. The semi-annual coupon rate for the bond is 7.5% convertible semi-annually. The bond is purchased to yield 9% convertible semi-annually.

   Calculate the principal in the coupon paid at the end of the 12th year.

   Solution:

   This is a question that can be done on your calculator.

$$\boxed{FV} \leftarrow 1300; \boxed{PMT} \leftarrow (1000)(0.075/2) = 37.50; \boxed{N} \leftarrow (20)(2) = 40; \boxed{I/Y} \leftarrow 9/2 = 4.5$$

$$\boxed{CPT}\boxed{PV}\boxed{2nd}\boxed{Amort}\boxed{P1} \leftarrow \boxed{P2} \leftarrow (12)(2) = 24\boxed{\downarrow}\boxed{\downarrow}\boxed{PRN} \rightarrow -9.94$$

2. Haozhe has 420,000 that he wants to invest. He has the following possible investments:

   a. Purchase a perpetuity immediate with annual payments for 420,000. The perpetuity will pay 1000 at the end of the first year, 2000 at the end of the second year, 3000 at the end of the third year,etc.

   b. Make a loan of 420,000 to Jun. Jun will repay the loan with level annual payments of 30,000 followed by a drop payment.

The annual effective interest rate for both investments is the same.

Calculate the amount of the drop payment.

Solution:

First determine the interest rate to be sued using Part a.

$$\frac{1000}{i}+\frac{1000}{i^2}=420,000 ==> 1000i+1000=420,000i^2 ==> i+1=420i^2$$

$$420i^2-i-1=0 ==> i=\frac{-(-1)\pm\sqrt{(-1)^2-(4)(420)(-1)}}{2(420)}=0.05$$

Now we find the drop payment:

$$\boxed{PV}\leftarrow 420,000;\boxed{I/Y}\leftarrow 5;\boxed{PMT}\leftarrow -30,000;\boxed{CPT}\boxed{N}\Rightarrow 24.676$$

$$\boxed{2nd}\boxed{Amort}\boxed{P1}\leftarrow 1;\boxed{P2}\leftarrow 24;\boxed{\downarrow}\boxed{Bal}\Rightarrow 19,482.01$$

$$Drop=(19,482.01)(1.05)=20,456.11$$

3. Kimberly purchases an 18 year continuous annuity that pays at a rate of $1300t$ at time t.

Calculate the present value of this annuity using a force of interest of $\delta = 0.0625$.

Solution:

$$1300(I\bar{a})_{\overline{18}|} = 1300\left[\frac{\frac{1-e^{-18(0.0625)}}{0.0625} - 18e^{-18(0.0625)}}{0.0625}\right] = 103,205.78$$

4. Tomas is receiving an annuity due with monthly payments for the next five years. The first payment is 500 at the start of the first month. The second payment is $500(1.08)$ at the beginning of the second month. The third payment is $500(1.08)^2$ at the beginning of the third month. The payments continue to increase in the same pattern.

   Calculate the present value of this annuity using an interest rate of 12% compounded monthly.

   Solution:

   Since payments ar monthly, we need $\frac{i^{(12)}}{12}$ which is $\frac{0.12}{12} = 0.01$

$$PV = 500 + 500(1.08)(1.01)^{-1} + ... + 500(1.08)^{59}(1.01)^{-59}$$

$$= \frac{500 - 500(1.08)^{60}(1.01)^{-60}}{1-(1.08)(1.01)^{-1}} = 394,887.72$$

5. Ram buys an annuity immediate for his Mom. The annuity will make quarterly payments to his Mom for 20 years. The payments are 1000 each quarter in the first year. The payments are 1100 each quarter of the second year. The payments continue to increase in the same pattern until payments of 2900 are paid each quarter of the 20th year.

   Using an interest rate of 8% compounded quarterly, calculate the price that Ram paid for this annuity. (The price is the present value of the payments.)

   Solutions:

   To solve this problem, we need to use the formula that does not follow the rules. However, since the first payment does not equal the amount of the increase, we must split the annuity into level payments of 900 and payments that are 100 the first year, 200 the second year, etc.

   To use the Formula that does not follow the rules, we need both $\frac{i^{(4)}}{4}$ and i.

$$\frac{i^{(4)}}{4} = \frac{0.08}{4} = 0.02 \qquad i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.02)^4 - 1 = 0.08243216$$

$$PV = 900a_{\overline{80}|0.02} + 100\left[\frac{\ddot{a}_{\overline{20}|0.08243216} - 20(1.08243216)^{-20}}{0.02}\right]$$

$$= 900\left[\frac{1-(1.02)^{-80}}{0.02}\right] + 100\left[\frac{\frac{1-(1.08243216)^{-20}}{0.08243216}(1.08243216) - 20(1.08243216)^{-20}}{0.02}\right]$$

$$= 67,448.36$$

6. The Huang Company invests 10,000 at the end of each year with DeWitt Bank. DeWitt Bank pays an annual effective interest rate of 6%.

   At the end of each year, Huang withdraws the interest earned from DeWitt and reinvests it in the Carvajal Fund which pays an annual effective interest rate of 8.5%.

   Determine the total amount that Huang has at the end of 15 years.

   Solution:

   The first year, there is no interest transfered to Carvajal as there is no money invested in DeWitt the first year. At the end of the second year, 600 is withdrawn from DeWitt and invested in Carvajal. At the end of the third year, 1200 is withdrawn from DeWittt and invested in Carvajal. This amount contines to increase each year.

   At the end of 15 years, there will be $(15)(10,000) = 150,000$ in the DeWitt Bank.

   At the end of 15 years, there will be

$$\left[600a_{\overline{14|}} + \frac{600}{i}\left(a_{\overline{14|}} - 14(1.085)^{-14}\right)\right](1.085)^{14} = 93,404.25$$

$$Total = 150,000 + 93,403.97 = 243,404.25$$

7. Xinyue has the choice of the following two bonds:

   a. Bond A is a 10 year par value bond with a maturity value of 10,000. The bond has a coupon rate of 8% convertible semi-annually.

   b. Bond B is a 10 year bond with a par value of F and a maturity value of $F+200$. The bond has a coupon rate of 7% convertible semi-annually.

Both bonds sell for a price of P to yield 7.5% convertible semi-annually. As a result, Bond B is purchased at a discount.

Determine the amount of the discount on Bond B.

Solution:

We use Part a to find the price P.

$$P = (10,000)(0.08/2)\left(\frac{1-(1.0375)^{-20}}{0.0375}\right)+(10,000)(1.0375)^{-20} = 10,347.41$$

Part b

$$P = 0.035F\left(\frac{1-(1.0375)^{-20}}{0.0375}\right)+(F+200)(1.0375)^{-20} = 10,347.41$$

$$\text{==>}\ (0.486367147)F + (0.478892342)F + 95.77846841 = 10,347.41$$

$$F = \frac{10,347.41-95.78}{0.486367147+0.478892342} = 10,620.59$$

$$C = F + 200 = 10,820.59$$

$$Discount = C - P = 10,820.59 - 10,347.41 = 473.18$$

8. Anunai has a loan that requires three annual payments to repay the loan. The interest rate on the loan is an annual effective interest rate of 6%.

   Complete the following amortization table. Show your work for full credit.

Time	Payment	Interest in Payment	Principle in Payment	Outstanding Loan Balance
0	---	---	---	$9000(1.06)^{-1}$ $+7000(1.06)^{-2}$ $+5000(1.06)^{-3}$ =18,918.64
1	9000	(18,918.64)(0.06) =1135.12	9000 – 1135.12 =7864.88	18,918.64 – 7864.88 =11,053.76
2	7000	(11,053.76)(0.06) =663.23	7000 – 663.23 =6336.77	11,053.76 – 6336.77 =4716.99
3	5000	(4716.99)(0.06) =283.02	5000 – 283.02 =4716.98	4716.99 – 4716.98 =0.01

9. Kovacik Corporation borrows an amount of L to be repaid under the sinking fund method. Each year for 15 years, Kovacik will pay the interest on the loan and make a deposit into the sinking fund. The amount to be paid into the sinking fund is such that the sinking fund will be equal to L at the end of 15 years.

The annual effective interest rate on the loan is 8% while the sinking fund will earn an annual effective interest rate of 6%.

At the end of 5 years, the amount in the sinking fund is 104,728.58.

Determine the amount of interest paid on the loan each year.

Solution:

$$D = \frac{L}{s_{\overline{15}|0.06}}$$

$$I = (L)(0.08)$$

$$\text{Sinking Fund Balance} = 104,728.58 = Ds_{\overline{5}|0.06}$$

$$D = \frac{104,728.58}{\left(\frac{(1.06)^5 - 1}{0.06}\right)} = 18,578.47$$

$$D = \frac{L}{s_{\overline{15}|0.06}} \text{ ==> } L = (18,578.47)\left(\frac{(1.06)^{15} - 1}{0.06}\right) = 432,431.98$$

$$I = (432,431.98)(0.08) = 34,594.56$$

10. David has a loan with n annual payments of 1000. The interest in the 15th payment is 799.02. The principle in the 20th payment is 246.89.

Find the outstanding loan balance right after the 30th payment.

Solution:

$$P_{15} = 1000 - 799.02 = 200.98$$

$$P_{15}(1+i)^{20-15} = P_{20} \text{ ==> } 200.98(1+i)^5 = 246.89 \text{ ==> } i = \left(\frac{246.89}{200.98}\right)^{01/5} - 1 = 0.042$$

$$OLB_{30}(0.042) = I_{31}$$

$$P_{31} = (246.89)(1.042)^{11} = 388.19 \text{ ==> } I_{31} = 1000 - 388.19 = 611.81$$

$$OLB_{30} = \frac{I_{31}}{0.042} = \frac{611.81}{0.042} = 14{,}566.90$$

There are other ways to do this which will get you a slightly different answer.

11. Jordan is the beneficiary of a 12 year continuous annuity. The annuity pays at a rate of $300+10t$ at time t.

Using a discount function of $1-0.04t$, calculate the present value of Jordan's annuity.

Solution:

$$PV=\int_0^{12} f(t)v(t)dt=\int_0^{12}(300+10t)(1-0.04t)dt=\int_0^{12} 300-12t+10t-0.4t^2\cdot dt=$$

$$\left[300t-t^2-\frac{0.4t^3}{3}\right]_0^{12}=3600-144-230.40=3225.60$$

12. Rahul buys a 20 year bond with semi-annual coupons. The maturity value of the bond is 100,000.

    The coupons increase. The first coupon is 200. The second coupon is 400. The third coupon is 600. The coupons continue to increase in the same pattern until the last coupon of 8000 is paid.

    The bond is bought to yield 10% convertible semi-annually.

    Determine the price of the bond.

    Solution:

    Price = PV of Cash Flows

$$= 200a_{\overline{40|}} + \frac{200}{0.05}\left(a_{\overline{40|}} - 46(1.05)^{-40}\right) + 100,000(1.05)^{-40}$$

$$= 200\left(\frac{1-(1.05)^{-40}}{0.05}\right) + \frac{200}{0.05}\left(\left(\frac{1-(1.05)^{-40}}{0.05}\right) - 40(1.05)^{-40}\right) + 100,000(1.05)^{-40}$$

$$= 63,545.42$$