MATH 373
Test 3
Fall 2018
December 10, 2018

1. A callable bond matures at the end of 20 years for its par value of 10,000. The bond pays coupons at a rate of 7% convertible semi-annually.

   The bond can be called at the end of 14 years for a call value of 10,500. The bond can be called at the end of 16 years for a call value of 10,350. Finally, the bond can be called at the end of 18 years for a call value of 10,200.

   Determine the price of this callable bond to yield a return of 6% convertible semi-annually.

   Solution:

N	I/Y	PMT	FV	CPT PV
(14)(2)=28	6%/2=3	(10,000)(0.07/2)=350	10,500	11,156.74
(16)(2)=32	3	350	10,350	11,155.36
(18)(2)=36	3	350	10,200	11,160.62
(20)(2)=40	3	350	10,000	11,155.74

   Price = 11,155.36

2. The stock of Widmer Corporation pays a quarterly dividend with the next dividend to be paid in two months. The next dividend paid in two months is expected to be 5. The second dividend paid in five months is expected to be 5(1.02). The third dividend paid in 8 months is expected to be $5(1.02)^2$. Dividends are expect to continue in the same pattern with each dividend being 102% of the previous dividend.

   Simon wants to buy the stock to yield 15% annually.

   Using the dividend discount method, determine the price of this stock.

   Solution:

   Since payments are made quarterly, we need $\frac{i^{(4)}}{4}$ and we have $i = 0.15$.

$$\frac{i^{(4)}}{4} = (1.15)^{0.25} - 1 = 0.035558$$

$$Price = PV = \left[5(1.035558)^{-1} + 5(1.02)(1.035558)^{-2} + 5(1.02)^2(1.035558)^{-3} + \ldots\right](1.035558)^{1/3}$$

$$= \left[\frac{5(1.035558)^{-1} - 0}{1 - (1.02)(1.035558)^{-1}}\right](1.035558)^{1/3} = 325.14$$

3. Lauren can purchase Ginuli Stock for 200 or Alex Stock for 200. Both stocks are expected to provide the same annual yield. Both prices are determined using the dividend discount method.

Ginuli Stock pays a level quarterly dividend of 5 with the next dividend paid later today.

Alex Stock is expected to pay an increasing dividend with the next dividend equal to D paid in three months. The second dividend paid in six months is expected to be $D+0.05$. The third dividend paid in nine months is expected to be $D+0.10$. Each dividend thereafter is expected to have the same pattern with the dividend being 0.05 greater than the previous dividend.

Determine D.

Solution:

$$\text{Let } i = \frac{i^{(4)}}{4}$$

$$Ginuili \Longrightarrow \left(\frac{5}{i}\right)(1+i) = 200 \Longrightarrow \frac{5}{i} + 5 = 200 \Longrightarrow i = \frac{5}{195}$$

$$Alex \Longrightarrow \frac{D}{i} + \frac{0.05}{i^2} = 200 \Longrightarrow \frac{D}{\left(\frac{5}{195}\right)} + \frac{0.05}{\left(\frac{5}{195}\right)^2} = 200 \Longrightarrow D = \left[200 - \frac{0.05}{\left(\frac{5}{195}\right)^2}\right]\left(\frac{5}{195}\right) = 3.18$$

4. An annuity due pays 1000 at the beginning of each year for 22 years.

Determine the Macaulay Duration of this annuity at an annual effective interest rate of 6%.

Solution:

$$MacDur = \frac{\sum C_t(t)v^t}{\sum C_t v^t} = \frac{(1000)(0)v^0 + (1000)(1)v^1 + \ldots + (1000)(21)v^{21}}{1000\ddot{a}_{\overline{22}|}}$$

$$\frac{(1000)v^1 + \ldots + (21,000)(21)v^{21}}{1000\ddot{a}_{\overline{22}|}} = \frac{1000a_{\overline{21}|} + \dfrac{1000}{0.06}(a_{\overline{21}|} - 21(1.06)^{-21})}{1000\ddot{a}_{\overline{22}|}} =$$

$$\frac{1000\left(\dfrac{1-(1.06)^{-21}}{0.06}\right) + \dfrac{1000}{0.06}\left(\dfrac{1-(1.06)^{-21}}{0.06} - 21(1.06)^{-21}\right)}{1000\left(\dfrac{1-(1.06)^{-22}}{0.06}\right)(1.06)} = 8.217$$

5. Tomas has agreed to pay the following payments to Taylen:

   a. 100,000 at the end of one year;

   b. 250,000 at the end of two years; and

   c. 400,000 at the end of four years.

   Tomas wants to exactly match the payments using the following bonds:

      i. Bond 1 is a one year bond with annual coupons of 100 and a maturity value of 1200.

      ii. Bond 2 is a two year bond with annual coupons of 80 and a maturity value of 1000.

      iii. Bond 3 is a three year bond with annual coupons of 400 and a maturity value of 10,000

      iv. Bond 4 is a zero coupon bond maturing for 100,000 at the end of four years.

   Determine the number of Bond 1 that Tomas should buy. Assume that you can buy partial bonds.

   Solution:

	Time 1	**Time 2**	**Time 3**	**Time 4**
Payments to Mary	100,000	250,000	0	400,000
Bond 1	1300			
Bond 2	80	1080		
Bond 3	400	400	14,000	
Bond 4	0	0	0	100,000

$$Bond4 ==> 100,000(Bond4) = 400,000 ==> Bond4 = 4$$

$$Bond3 ==> 10,400(Bond3) = 0 ==> Bond3 = 0$$

$$Bond2 ==> 400(Bond3) + 1080(Bond2) = 250,000$$

$$==> 400(0) + 1080(Bond2) = 250,000 ==> Bond2 = \frac{250,000}{1080} = 231.48148$$

$$Bond1 ==> 400(Bond3) + 80(Bond2) + 1300(Bond1) = 100,000$$

$$==> 400(0) + 80(231.48148) + 1300(Bond1) = 100,000$$

$$Bond1 = \frac{100,000 - 80(231.48148)}{1300} = 62.678$$

6. Tomas has agreed to pay the following payments to Taylen:

    a. 100,000 at the end of one year;

    b. 250,000 at the end of two years; and

    c. 400,000 at the end of four years.

Calculate the Modified Convexity of these payments at an interest rate of 8%.

Solution:

$$ModCon = v^2 \frac{\sum C_t (t)(t+1)v^t}{\sum C_t v^t}$$

$$= (1.08)^{-2}\left(\frac{(100,000)(1)(2)(1.08)^{-1} + (250,000)(2)(3)(1.08)^{-2} + (400,000)(4)(5)(1.08)^{-4}}{(100,000)(1.08)^{-1} + (250,000)(1.08)^{-2} + (400,000)(1.08)^{-4}} \right)$$

$$= 10.488$$

7. The Bray Insurance Company has the following portfolio of annuities that it will be paying over the next several years:

	Present Value of Future Payments	**Modified Duration**
Annuity 1	400,000	8
Annuity 2	350,000	10
Annuity 3	250,000	14

Christine who is the actuary for Bray, wants to spend 1,000,000 to purchase bonds. Christine can purchase the following two bonds in any amount. (In other words, she can purchase partial bonds.)

	Modified Duration
Bond 1	6
Bond 2	18

Christine's objective in buying bonds is to match the Modified Duration of the annuity portfolio.

Determine the amount that Christine should spend on Bond 1.

Solution:

For the annuities:

$$D_{ModDur}^{Port} = \frac{(400,000)(8)+(350,000)(10)+(250,000)(14)}{400,000+350,000+250,000} = 10.2$$

For the Assets:

$$D_{ModDur}^{Port} = \frac{(X)(6)+(1,000,000-X)(18)}{1,000,000} = 10.2$$

$$10,200,000 = 18,000,000 - 12X$$

$$X = \frac{18,000,000-10,200,000}{12} = 650,000$$

8. Tong LTD owns two bonds. The bonds have the following characteristics:

	Price	Modified Convexity	Macaulay Convexity
Bond A	40,000	58	60
Bond B	60,000	86	90

The above values are all based on an annual effective interest rate of 8%.

Using the First Order Macaulay Approximation, calculate the estimated price of this bond portfolio if the annual effective interest rate is 7%.

Solution:

$$P(i) = P(i_0)\left[\frac{1+i_0}{1+i}\right]^{MacDur}$$

$$P(i_0) = 40,000 + 60,000 = 100,000$$

$$i = 0.07 \text{ and } i_0 = 0.08$$

$$ModCon = v^2(MacDur + MacCon) \text{ ==> } 58 = (1.08)^{-2}(MacDur + 60)$$

$$\text{==> } MacDur \text{ Bond A=7.6512}$$

$$ModCon = v^2(MacDur + MacCon) \text{ ==> } 86 = (1.08)^{-2}(MacDur + 90)$$

$$\text{==> } MacDur \text{ Bond B=10.3104}$$

$$D_{MacDur}^{Port} = \frac{(40,000)(7.6512) + (60,000)(10.3104)}{100,000} = 9.2467$$

$$P(i) = (100,000)\left[\frac{1.08}{1.07}\right]^{9.2467} = 108,982.42$$

9. Sally wants to fully immunize a future payment of X at time Y using the following two bonds:
   a. Bond A is a zero coupon bond maturing in 4 years; and
   b. Bond B is a zero coupon bond maturing in 17 years.

Sally pays 231,933.38 for Bond A and 77,311.13 for Bond B.

Determine X and Y if the annual effective interest rate of 7%.

Solution:

PV of Assets = PV of Liabilities = 231,933.38+77,311.13 = 309,244.21

Duration of Liability is Y. It must equal duration of Assets.

$$==> \frac{4(213,933.38)+(17)(77,311.13)}{309,244.21} = Y ==> Y = 7.25$$

$$PV \text{ of Assets = PV of Liabilities } ==> 309,244.21 = X(1.07)^{-7.25}$$

$$==> X = 505,050$$

10. Jake will pay Yash 100 at the end of one year, 200 at the end of two years, 300 at the end of three years, and 400 at the end of four years.

    Calculate the Modified Duration of these payments at an interest rate of 10%.

    Solution:

$$ModDur = v\frac{\sum C_t(t)v^t}{\sum C_t v^t}$$

$$= (1.10)^{-1}\left(\frac{(100)(1)(1.10)^{-1} + (200)(2)(1.10)^{-2} + (300)(3)(1.10)^{-3} + (400)(4)(1.10)^{-4}}{(100)(1.10)^{-1} + (200)(1.10)^{-2} + (300)(1.10)^{-3} + (400)(1.10)^{-4}}\right)$$

$$= 2.638$$

11. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	4.00%	3.0	5.85%
1.0	4.50%	3.5	6.20%
1.5	4.95%	4.0	6.35%
2.0	5.30%	4.5	6.45%
2.5	5.60%	5.0	6.50%

Determine the price of a 2 year bond with semi-annual coupons of 200 and a maturity value of 3000.

Solution:

$$P = PV = (200)(1.04)^{-0.5} + (200)(1.045)^{-1} + (200)(1.0495)^{-1.5} + 3200(1.053)^{-2}$$

$$= 3459.50$$

12. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	4.00%	3.0	5.85%
1.0	4.50%	3.5	6.20%
1.5	4.95%	4.0	6.35%
2.0	5.30%	4.5	6.45%
2.5	5.60%	5.0	6.50%

Tianjian invested 10,000 today and another 10,000 at the end of two years.

How much will Tianjian have at the end of four years?

Solution:

First, find the present value using the spot rates.

$$PV = 10,000 + 10,000(1.053)^{-2} = 19,018.68582$$

Then $AV = PV(1.0635)^4 = 24,329.35$

13. You can buy the following three bonds:

    i. A six month zero coupon bond with a maturity value of 1000 and a price of 970.

    ii. A one year bond with semi-annual coupons of 100 and a maturity value of 1000. The price of the bond is 1130.

    iii. A bond that matures in 18 months with semi-annual coupons of 400 and a maturity value of 800. The price of this bond is 1845.

Determine the spot rate for 18 months.

Solution:

Using Bond i:

$$970(1+r_{0.5})^{0.5} = 1000 \implies r_{0.5} = 0.062812201$$

Using Bond ii:

$$1130 = 100(1+r_{0.5})^{-0.5} + 1100(1+r_1)^{-1} \implies r_1 = \frac{1100}{1130 - 100(1.062812201)^{-0.5}} - 1 = 0.064859632$$

Using Bond iii:

$$1845 = 400(1+r_{0.5})^{-0.5} + 400(1+r_1)^{-1} + 1200(1+r_{1.5})^{-1.5}$$

$$1845 = 400(1.062812201)^{-0.5} + 400(1.064859632)^{-1} + 1200(1+r_{1.5})^{-1.5}$$

$$r_{1.5} = \left(\frac{1200}{1845 - 400(1.062812201)^{-0.5} - 400(1.064859632)^{-1}} \right)^{1/1.5} - 1 = 0.071864$$

You are given the following spot interest rates and information for questions 14 and 15:

t	r_t	t	r_t
0.5	4.00%	3.0	5.85%
1.0	4.50%	3.5	6.20%
1.5	4.95%	4.0	6.35%
2.0	5.30%	4.5	6.45%
2.5	5.60%	5.0	6.50%

Jinks Corporation has a four year loan from Anderson Bank for an amount of 400,000. The loan has a variable interest rate which is equal to the one year spot rate at the beginning of each year of the loan. Jinks will pay the interest on the loan at the end of each year for four years. Additionally, Jinks will pay the principal of the loan which is 400,000 at the end of the fourth year.

Jinks would like to have a fixed interest rate instead of a variable interest rate. Therefore, Jinks enters into an interest rate swap with Lai Investment Bank. Under the swap, Jinks will pay a fixed interest rate to Lai and Lai will pay the variable interest rate to Jinks. The terms of the swap mirror the terms of the loan.

14. Answer the following four questions which will count as one question on the exam:

    a. Who is the receiver in this scenario?

       Lai Investment Bank

    b. Who is the payer in this scenario?

       Jinks Corporation

    c. State the Settlement Period for this Swap.

       One Year

    d. State the Notional Amount for this Swap.

       400,000

15. Determine the Swap Rate for this Swap.

$$R = \frac{1 - P_4}{P_1 + P_2 + P_3 + P_4} = \frac{1 - (1.0635)^{-4}}{(1.045)^{-1} + (1.053)^{-2} + (1.0585)^{-3} + (1.0635)^{-4}} = 0.06266$$

16. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	4.00%	3.0	5.85%
1.0	4.50%	3.5	6.20%
1.5	4.95%	4.0	6.35%
2.0	5.30%	4.5	6.45%
2.5	5.60%	5.0	6.50%

Shina buys a deferred interest rate swap. The Swap Tenor is five years. There is no interest rate swap for the first three years. During the last two years, Shina swaps a variable interest rate for a fixed interest rate. The variable interest rate is equal to the one year spot rate at the beginning of each year. The swap is based on a notional amount of 500,000 during the 4th year and 1,000,000 during the last year.

Determine the Swap Rate.

Solution:

$$R = \frac{\sum Q \cdot f \cdot P}{\sum Q \cdot P}$$

$$(1.0585)^3(1+f_{[3,4]}) = (1.0635)^4 \implies f_{[3,4]} = \frac{(1.0635)^4}{(1.0585)^3} - 1 = 0.078642157$$

$$(1.0635)^4(1+f_{[4,5]}) = (1.0650)^5 \implies f_{[4,5]} = \frac{(1.065)^5}{(1.0635)^4} - 1 = 0.071021186$$

$$R = \frac{(500,000)(0.078642157)(1.0635)^{-4} + (1,000,000)(0.071021186)(1.065)^{-5}}{(500,000)(1.0635)^{-4} + (1,000,000)(1.065)^{-5}} = 0.073689$$

The following information is provided for Questions 17 and 18:

On January 1, **2015**, Joe entered into a five year interest rate swap with swap periods of one year. The swap has a notional amount of 300,000. Under the swap, Joe will pay a fixed rate of 6.5% at the end of each year and will receive a payment based on the variable interest rate which is equal to the one year spot rate at the beginning of each year.

On January 1, **2018**, the spot interest rates are as follows:

t	r_t
0.5	5.80%
1.0	6.20%
1.5	6.75%
2.0	7.00%
2.5	7.20%

17. Calculate the net swap payment for Joe at the end of the fourth year of the Swap. (Be sure to state whether Joe will make the payment or receive the payment.)

Solution:

Joe will pay fixed rate ==> $(300,000)(0.065) = 19,500$

Joe will receive variable rate ==> $(300,000)(0.062) = 18,600$

$19,500 - 18,600 = 900$

Joe will make a payment of 900.

18. Joe decides to sell the swap at the start of the fourth year of the swap.

Determine the market value of the swap from Joe's viewpoint.

Solution:

Joe will pay fixed rate ==> $(300,000)(0.065) = 19,500$. He will pay this at the end of the fourth year and at the end of the fifth year.

Joe will receive variable rate ==> $(300,000)(0.062) = 18,600$ at the end of the fourth year. He will also receive the variable rate at the end of the fifth year. We base that calculation on the forward rate = $\frac{(1.07)^2}{1.062} - 1 = 0.07806$

==> $(300,000)(0.07806) = 23,418$ is the amount Joe will receive at the end of 5th year.

Market Value = Present Value that Joe will receive less Present Value that Joe will pay

$$= \frac{18,600 - 19,500}{1.062} + \frac{23,418 - 19,500}{(1.07)^2} = 2574.68$$