MATH 373
Quiz 1
Fall, 2019

September 10, 2019

1. Deepa borrows 1000 from Ally. Deepa agrees to pay a nominal interest rate of 9% compounded quarterly.

   At the end of X years, Deepa repays the loan with a payment of 1427.62.

   Calculate X.

   Solution:

   $i^{(4)} = 9\%$ $\qquad$ $\frac{i^{(4)}}{4} = \frac{9\%}{4} = 2.25\%$

   $(1+i)^X = (1+\frac{i^{(4)}}{4})^{4X}$

   $1000(1.0225)^{4X} = 1427.62$ $\quad$ ==> $\quad$ $(1.0225)^{4X} = 1.42762$

   $4X = \frac{\ln(1.42762)}{\ln(1.0225)} = 16$ $\quad$ ==> $\quad$ $X = 4$

   OR

   Using the Calculator:

   $\boxed{I/Y} = 2.25\%$
   $\boxed{PV} = 1000$
   $\boxed{FV} = -1427.62$
   $\boxed{CPT}\boxed{N} = 16$

   $N = 4X \Leftrightarrow X = \frac{N}{4}$

   $N = 16 \Rightarrow X = 4$

2. You are given that $i = 8\%$.

   Let $i^{(4)}$ be the nominal annual interest rate compounded quarterly that is equivalent to i.

   Let $d^{(12)}$ be the nominal annual discount rate compounded monthly that is equivalent to i.

   Calculate $(1000)(i^{(4)} - d^{(12)})$.

   Solution:

$$(1+i) = 1.08 = \left(1 + \frac{i^{(4)}}{4}\right)^4 \Rightarrow i^{(4)} = 0.077706188$$

$$(1+i) = 1.08 = \left(1 - \frac{d^{(12)}}{12}\right)^{-12} \Rightarrow d^{(12)} = 0.076714776$$

$$\begin{aligned} 1000(i^{(4)} - d^{(12)}) &= 1000(0.077706188 - 0.076714776) \\ &= 0.991412 \end{aligned}$$

3. Britney invests 10,000 in an account earning interest based on an accumulation function of $\alpha + \beta t^2$. After 3 years, Britney has 11,800.

   Determine the effective interest rate that Britney earned during the third year. This would be $i_{[2,3]}$ in symbols.

   Solution:

$$a(t) = \alpha + \beta t^2$$

$$a(0) = 1 = \alpha + \beta(0^2) \Rightarrow \alpha = 1$$

$$(10,000)a(3) = 10,000(\alpha + \beta(3^2)) = 11,800$$

$$11,800 = 10,000 + 90,000\beta$$

$$1,800 = 90,000\beta \Rightarrow \beta = 0.02$$

$$\begin{aligned} i_{[2,3]} &= \frac{a(3) - a(2)}{a(2)} \\ &= \frac{(1 + 0.02(3^2)) - (1 + 0.02(2^2))}{(1 + 0.02(2^2))} \\ &= 0.092592593 \end{aligned}$$