MATH 373
Quiz 5
Fall 2019

November 21, 2019

1. You are given the following spot interest rate curve:

t	r_t
0.5	0.020
1.0	0.026
1.5	0.031
2.0	0.035
2.5	0.039
3.0	0.042
3.5	0.045
4.0	0.048
4.5	0.051
5.0	0.054

Use the spot interest rates to calculate the accumulated value of an annuity due with payments of 24,000 at the beginning of each year for 3 years.

Solution:

First, we find the present value, then we find the accumulated value.

$$PV = 24,000(1+(1+r_1)^{-1}+(1+r_2)^{-2})$$

$$= 24,000(1+(1.026)^{-1}+(1.035)^{-2})$$

$$= 69,796.07$$

$$AV = PV(1+r_3)^3 = 69,796.07(1.042)^3 = 78,964.91$$

2. You are given the following spot interest rate curve:

t	r_t
0.5	0.020
1.0	0.026
1.5	0.031
2.0	0.035
2.5	0.039
3.0	0.042
3.5	0.045
4.0	0.048
4.5	0.051
5.0	0.054

Use the spot interest rates to calculate the price of two year par value bond with a maturity value of 10,000. The bond pays semi-annual coupon at a rate of 8% convertible semi-annually.

Solution:

$$\text{Semi-annual Coupon } = Fr = 10,000\left(\frac{0.08}{2}\right) = 400$$

$$Price = PV = 400(1+r_{0.5})^{-0.5} + 400(1+r_1)^{-1} + 400(1+r_{1.5})^{-1.5} + 10,400(1+r_2)^{-2}$$

$$= 400(1.02)^{-0.5} + 400(1.026)^{-1} + 400(1.031)^{-1.5} + 10,400(1.035)^{-2}$$

$$= 10,876.53$$

3. You are given the following three bonds:

   a. A one year bond which sells for 990 and has a maturity value of 1000 and annual coupons of 42.

   b. A two year bond with a maturity value of 50,000 and annual coupons of 10,000. The price of this bond is 61,000.

   c. A three year zero coupon bond with a maturity value of 100,000 which sells for 77,000.

   Use bootstrapping to find $f_{[1,3]}$.

   Solution:

   First, we need to find the spot rates using bootstrapping. Then, using the spot rates, we will find the forward rate.

$$Price\ of\ A = 990 = \frac{1042}{1+r_1} \quad ==> \quad r_1 = \frac{1042}{990} - 1 = 0.052525253$$

$$Price\ of\ B = 61{,}000 = \frac{10{,}000}{1+r_1} + \frac{60{,}000}{(1+r_2)^2}$$

$$\frac{60{,}000}{(1+r_2)^2} = 61{,}000 - \frac{10{,}000}{1.052525253} = 51{,}499.04 \Rightarrow r_2 = \left(\frac{60{,}000}{51{,}499.04}\right)^{\frac{1}{2}} - 1 = 0.079384202$$

$$Price\ of\ C = 77{,}000 = \frac{10{,}000}{(1+r_3)^3} \quad ==> \quad r_3 = \left(\frac{100{,}000}{77{,}000}\right)^{\frac{1}{3}} - 1 = 0.091029328$$

$$(1+r_1)(1+f_{[1,3]})^2 = (1+r_3)^3 \quad ==> \quad f_{[1,3]} = \left(\frac{(1+r_3)^3}{1+r_1}\right)^{\frac{1}{2}} - 1$$

$$f_{[1,3]} = \left(\frac{(1.091029328)^3}{1.052525253}\right)^{\frac{1}{2}} - 1 = 0.110806405$$