

MATH 373

Test 1

Fall 2019

October 1, 2019

1. John invests 1000 in Anderson Bank. Based on the Rule of 72, John expects to have 2000 at the end of 18 years.

How much will John actually have at the end of 18 years?

Solution:

Using the Rule of 72, money doubles in $\frac{72}{i}$ years.

$$\frac{72}{i} = 18 \implies i = \frac{72}{18} \% = 4\% = 0.04$$

$$1000(1+0.04)^{18} = 1000(1.04)^{18} = 2025.82$$

2. Brenna wants to have 500,000 on her 40th birthday. Today is her 20th birthday. She will make a deposit of P into an account at the beginning of each month for the next 20 years. The account will earn an annual effective interest rate of 8%.

Determine P so that Brenna will have exactly 500,000 on her 40th birthday.

Solution:

Since payments are monthly, we need the monthly effective interest rate of $\frac{i^{(12)}}{12}$.

$$\left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1 + i = 1.08 \implies \frac{i^{(12)}}{12} = (1.08)^{\frac{1}{12}} - 1 = 0.006434$$

$$P\ddot{s}_{\overline{240}|} = 500,000 \implies P \left(\frac{(1.006434)^{240} - 1}{0.006434} \right) (1.006434) = 500,000$$

$$\implies 572.66P = 500,000 \implies P = 873.12$$

3. Alan invests 10,000 in an account earning simple interest. At the end of 20 years, Alan has 30,000.

Kelly invests 10,000 in an account earning compound interest at an annual effective interest rate of i .

During the 11th year, Alan and Kelly earn the same annual effective interest rate.

Determine the amount that Kelly has at the end of the 20th year.

Solution:

Alan:

$$10,000(1 + 20s) = 30,000 \implies 10,000 + 200,000s = 30,000$$

$$\implies 1 + 20s = 3 \implies 20s = 2 \implies s = 0.1$$

$$i_{11}^{Alan} = i_{11}^{Kelly}$$

$$i_{11}^{Alan} = \frac{s}{1 + (n-1)s} = \frac{0.1}{1 + (11-1)(0.1)} = 0.05$$

$$i_{11}^{Kelly} = i = 0.05$$

$$Amount = (10,000)(1 + 0.05)^{20} = 26,532.98$$

4. Seth loans 5000 to Ram. Ram will repay the loan with the following payments:

| Time | Payment |
|------|---------|
| 2 | 2700 |
| 4 | 2700 |

Seth takes each payment and reinvests it at an annual effective interest rate of r . Taking into account reinvestment, Seth realizes an annual effective return on the loan of 3.74%.

Determine r

Solution:

$$5000(1.0374)^4 = 2700(1+r)^2 + 2700$$

$$5791.02 = 2700(1+r)^2 + 2700$$

$$3091.02 = 2700(1+r)^2$$

$$(1+r)^2 = 1.14482$$

$$(1+r) = 1.069963$$

$$r = 0.069963 = 6.9963\%$$

5. Alberto, Brian, and Vanessa enter into a financial agreement. Alberto agrees to pay Brian 10,000 today. Alberto will also pay 20,000 to Vanessa today.

At the end of one year, Brian will pay Vanessa 7,000.

At the end of two years, Vanessa will pay 20,000 to Alberto.

At the end of four years, Vanessa will pay 14,000 to Alberto. Additionally, at the end of four years, Brian will pay 5,723 to Alberto.

Finally, at the end of 5 years, Vanessa pays X to Brian.

Over the five year period, the annual effective interest rate paid or received by each party is the same.

Determine X .

Solution:

Alberto's cashflows are -30,000 at time 0, 20,000 at time 2, 19,723 at time 4.

His equation of value is:

$$30,000(1+i)^4 = 20,000(1+i)^2 + 19,723$$

$$\text{Let } x = (1+i)^2$$

$$30,000x^2 = 20,000x + 19,723 \implies 30,000x^2 - 20,000x - 19,723 = 0$$

$$x = \frac{-(-20,000) \pm \sqrt{(-20,000)^2 - 4(30,000)(-19,723)}}{2(30,000)} = 1.21$$

$$x = (1+i)^2 = 1.21 \implies i = (1.21)^{\frac{1}{2}} - 1 = 0.1$$

(cont. below)

Brian's cashflows are 10,000 at time 0, -7,000 at time 1, -5,723 at time 4, X at time 5.

$$10,000(1.1)^5 + X = 7,000(1.1)^4 + 5,723(1.1)$$

$$X = 7,000(1.1)^4 + 5,723(1.1) - 10,000(1.1)^5 = 438.90$$

OR

Vanessa's cashflows are 20,000 at time 0, 7,000 at time 1, -20,000 at time 2, -14,000 at time 4, -X at time 5.

$$20,000(1.1)^5 + 7,000(1.1)^4 = 20,000(1.1)^3 - 14,000(1.1) + X$$

$$X = 20,000(1.1)^5 + 7,000(1.1)^4 - 20,000(1.1)^3 - 14,000(1.1) = 438.90$$

$$X = 438.90$$

6. Abishek borrows 25,000 to buy a car. Abishek will repay the loan with 60 monthly payments. The interest rate on the loan is 9% compounded monthly.

Abishek makes the first 30 payments on time. Additionally, at the end of the 15th month, he makes an additional payment of 1000 over and above the normal payment.

Determine the outstanding loan balance right after the 30th payment.

Solution:

The monthly effective interest rate is: $\frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075$.

$$Pa_{\overline{60}|} = 25,000 = P \left(\frac{1 - (1.0075)^{-60}}{0.0075} \right) \implies 25,000 = 48.17P \implies P = 518.96$$

OLB_{30} = Accumulated Value of Past Cash Flows

$$\begin{aligned} OLB_{30} &= 25,000(1.0075)^{30} - 518.96s_{\overline{30}|} - 1000(1.0075)^{15} \\ &= 25,000(1.0075)^{30} - 518.96 \left(\frac{(1.0075)^{30} - 1}{0.0075} \right) - 1000(1.0075)^{15} \\ &= 31,281.79 - 17,386.63 - 1,118.60 \\ &= 12,776.56 \end{aligned}$$

7. Christopher has won the lottery. He has the choice of the following three options:
- A lump sum of 10,000,000
 - A perpetuity due of with annual payments of 740,740.74.
 - An annuity with 20 payments at the beginning of each year for 20 years. The first payment is P . The second payment is $P(1.2)$. The third payment is $P(1.2)^2$. The payments continue in the same pattern with each payment being 120% of the prior payment.

All payments have the same present value at an annual interest rate of i .

Determine P .

Solution:

$$10,000,000 = \frac{740,740.74}{i}(1+i)$$

$$10,000,000i = 740,740.74 + 740,740.74i$$

$$9,259,259i = 740,740.74 \implies i = 0.08$$

$$v = (1.08)^{-1}$$

$$\begin{aligned} 10,000,000 &= \frac{P - P(1.2)^{20}v^{20}}{1 - 1.2v} \\ &= P \left(\frac{1 - (1.2)^{20}(1.08)^{-20}}{1 - (1.2)(1.08)^{-1}} \right) \end{aligned}$$

$$10,000,000 = 65.027P \implies P = 153,781.40$$

8. Varun invests 10,000 in an account today.

During the first 5 years, the account earns an interest rate of 6% compounded quarterly.

During the second 5 years, the account earns a rate equivalent to a discount rate of 12% convertible monthly.

Determine the amount that Varun has at the end of 10 years.

Solution:

$$\begin{aligned} 10,000 \left(1 + \frac{0.06}{4} \right)^{4(5)} \left(1 - \frac{0.12}{12} \right)^{-12(5)} &= 10,000(1.015)^{20} (0.99)^{-60} \\ &= 24,615.53 \end{aligned}$$

9. Sid wants to provide an increasing annuity for his mother for the next 20 years. The annuity will make annual payments at the beginning of each year. The first payment today will be 1000. The second payment will be 3000. The third payment will be 5000. The payments will continue in the same pattern each payment being 2000 greater than the prior payment.

Using an annual effective interest rate of 6.25%, calculate the present value of the annuity.

Solution:

$$\begin{aligned} PV &= \left[1000a_{\overline{20}|} + \left(\frac{2000}{0.0625} \right) \left(a_{\overline{20}|} - 20(1.0625)^{-20} \right) \right] (1.0625) \\ &= \left[1000 \left(\frac{1 - (1.0625)^{-20}}{0.0625} \right) + \left(\frac{2000}{0.0625} \right) \left(\frac{1 - (1.0625)^{-20}}{0.0625} - 20(1.0625)^{-20} \right) \right] (1.0625) \\ &= 191,858.39 \end{aligned}$$

We multiply the P&Q formula above by (1.0625) since this is an annuity due.

10. Claire invests 20,000 today in an account earning a force of interest of $\delta_t = 0.02 + 0.01t$.

How much does Claire have at the end of 10 years.

Solution:

$$\begin{aligned} \text{Amount} &= 20,000e^{\int_0^{10} (0.02+0.01t)dt} \\ &= 20,000e^{0.02t+0.005t^2}\Big|_0^{10} \\ &= 20,000e^{0.7} \\ &= 20,000(2.01375) \\ &= 40,275.05 \end{aligned}$$

11. Ally invests money in the Ginuli Fund for two years. She wants to approximate her annual dollar weighted return over those two years.

Ally initially invests 5,000 into the Fund. At the end of 12 months, Ally has 6,000 and deposits 4000. At the end of 19 months, Ally has 10,500 and withdraws 2500. After 24 months, Ally has 9,000.

Estimate Ally's annual dollar weighted rate using the simple interest method that we learned in class.

Solution:

$$A = 5,000; B = 9,000; C = 4,000 - 2,500 = 1,500$$

$$A + C + I = B \implies 5,000 + 1,500 + I = 9,000 \implies I = 2,500$$

$$j = \frac{2,500}{5,000 + 4,000 \left(1 - \frac{12}{24}\right) - 2,500 \left(1 - \frac{19}{24}\right)} = 0.38585$$

$$1 + i = (1 + j)^{\frac{1}{T}} = (1.38585)^{\frac{1}{2}} = 1.17722 \implies i = 0.17722$$

12. Thomas Corporation invests 10 million to build a new factory. In return, he expects to receive a payment of 6 million at the end of 2 years and 8 million at the end of 4 years.

Determine the Internal Rate of Return on Thomas' investment.

Solution:

$$-10,000,000 + 6,000,000(1+i)^{-2} + 8,000,000(1+i)^{-4} = 0$$

$$\text{Let } x = (1+i)^{-2}$$

$$-10,000,000 + 6,000,000x + 8,000,000x^2 = 0 \implies -10 + 6x + 8x^2 = 0$$

$$x = \frac{-6 + \sqrt{6^2 - 4(8)(-10)}}{2(8)} = 0.80425$$

$$(1+i)^{-2} = 0.80425 \implies i = 0.11508 = 11.508\%$$

OR

Using the BA-II+ Calculator:

$$CF0 = -10,000,000$$

$$C01 = 0; F01 = 1$$

$$C02 = 6,000,000; F02 = 1$$

$$C03 = 0; F03 = 1$$

$$C04 = 8,000,000; F04 = 1$$

$$\boxed{CPT} \boxed{IRR} \rightarrow 11.508\%$$