MATH 373
Test 2
Fall 2019

October 31, 2019

1. Ally buys a 10 year callable bond. The bond matures at the end of 10 years for 10,000. The bond has semi-annual coupons at a rate of 7.5% convertible semi-annually.

   The bond can be called at the end of 5 years for a call value of 10,225.

   The bond can be called at the end of 7 years for a call value of 10,125.

   The bond can be called at the end of 9 years for a call value of 10,050.

   The bond is purchased to yield 7% convertible semi-annually.

   Determine the price of the bond.

   Solution:

   Calculate the value at each call date and the maturity date and select the lowest price.

N	I/Y	PMT	FV	CPT PV
10	3.5	(10,000)(0.0375)=375	10,225	10,367.42
14	3.5	375	10,125	10,350.24
18	3.5	375	10,050	10,356.66
20	3.5	375	10,000	10,355.31

   The lowest present value is the price → 10,350.24

2. Peggy is receiving an annuity immediate with monthly payments for the next 20 years. The first payment is 100. The second payment is 200. The third payment is 300. The payments continue increase until the last payment is 24,000.

   Peggy takes each payment and invests it in the Tang Fund. The Tang Fund pays an interest rate of 9% compounded monthly.

   Determine the amount that Peggy will have at the end of 20 years.

   Solution:

   Every payment is increasing so we have the P&Q formula.

$$i^{(12)} = 0.09 \Rightarrow \frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075$$

$$\begin{aligned} AV &= \left[100a_{\overline{240|}} + \frac{100}{0.0075}\left(a_{\overline{240|}} - 240v^{240}\right)\right](1.0075)^{240} \\ &= \left[100\left(\frac{1-(1.0075)^{-240}}{0.0075}\right) + \frac{100}{0.0075}\left(\left(\frac{1-(1.0075)^{-240}}{0.0075}\right) - 240(1.0075)^{-240}\right)\right](1.0075)^{240} \\ &= (960,526.11)(1.0075)^{240} \\ &= 5,771,946.95 \end{aligned}$$

3. Claire is receiving a 25 year continuous annuity with payments of $1234t$ at time t.

Using a force of interest of 6%, calculate the present value of Claire's annuity.

Solution:

$$\int_0^{25} (1234t)v^t dt = 1234\left(\bar{I}\bar{a}\right)_{\overline{25|}}$$

$$= 1234\left(\frac{\dfrac{1-e^{-(25)(0.06)}}{0.06} - 25e^{-(25)(0.06)}}{0.06}\right)$$

$$= 1234(122.83)$$

$$= 151{,}567.63$$

4. Josh has a 10 year loan which is being repaid with non-level annual payments. The first payment is 10,000. The second payment is 9000. The third payment is 8000. The payments continue to decrease until the last payment is 1000.

   The loan has an annual effective interest rate of 6%.

   Determine the principal in the payment of 3000 at the end of the 8^{th} year.

   Solution:

   To find the principal in the 8^{th} payment, we need to find the OLB after the 7^{th} payment.

$$OLB_7 = 3000(1.06)^{-1} + 2000(1.06)^{-2} + 1000(1.06)^{-3}$$

$$= 5449.80$$

$$I_8 = (5449.80)(0.06) = 326.99$$

$$P_8 = PMT_8 - I_8 = 3000 - 326.99 = 2673.01$$

5. Alberto wants to borrow 100,000. He approaches two banks who offer the following loans:

   a. The Mills Bank offers an amortization loan to be repaid over 10 years with level annual payments. The interest rate on the loan is an annual effective interest rate of 6.5%.

   b. The Bank of Ross offers a 10 year sinking fund loan. Under the sinking fund loan, Alberto will need to pay the interest on the loan at an annual interest rate of 6% to the Bank at the end of each year. Additionally, Alberto will need to make a sinking fund deposit at the end of each year so that the amount in the sinking fund at the end 10 years is 100,000. The sinking fund will earn an annual effective interest rate of 5%.

State which loan Alberto should accept and demonstrate why.

Solution:

Option a:

Use your calculator to find the annual payment:

$$\boxed{N} \leftarrow 10; \boxed{I/Y} \leftarrow 6.5; \boxed{PV} \leftarrow -100,000$$

$$\boxed{CPT}\boxed{PMT} \Rightarrow 13,910.47$$

OR

$$PMT = \frac{100,000}{a_{\overline{10}|}} = \frac{100,000}{\frac{1-(1.065)^{-10}}{0.065}} = 13,910.47$$

Option b:

$$I = (100,000)(0.06) = 6000$$

$$D = \frac{100,000}{s_{\overline{10}|}} = \frac{100,000}{\frac{(1.05)^{10}-1}{0.05}} = 7950.46$$

Total Payment $= 6000 + 7950.46 = 13,950.46$

Alberto should select Option a because the total annual payment is smaller.

6. Kelley Korporation borrows 500,000 at an annual effective interest rate of 7.5%. Kelley will repay the loan with level annual payments of 59,000 plus a balloon payment.

   Determine the amount of the balloon payment.

   Solution:

   Use your calculator to find n:

   $\boxed{PV} \leftarrow 500,000; \boxed{I/Y} \leftarrow 7.5; \boxed{PMT} \leftarrow -59,000$

   $\boxed{CPT}\boxed{N} \rightarrow 13.958 \Rightarrow$ round down to 13

   $\boxed{2nd}\boxed{Amort}\boxed{P1} \leftarrow 1; \boxed{P2} \leftarrow 13; \boxed{\downarrow}\boxed{Bal} \rightarrow 52,681.5869$

   $Balloon = Bal + Q = 52,681.59 + 59,000 = 111,681.59$

7. A 20 year bond has non-level coupons which are paid semi-annually. The bond matures for 100,000. Each coupon in the first year is 300. Each coupon in the second year is 600. Each coupon in the third year is 900. The coupons continue to increase in the same pattern until each coupon in the 20 year is 6000.

   The bond is bought to yield 6% compounded semi-annually.

   Determine the price of the bond.

   Solution:

   The price is the present value of cash flows. We have to use the Formula That Does Not Follow The Rules since the payments are level during the year but increase year to year.

$$i^{(2)} = 0.06 \Rightarrow \frac{i^{(2)}}{2} = \frac{0.06}{2} = 0.03$$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1 = (1.03)^2 - 1 = 0.0609$$

$$PV = 300\left(\frac{\ddot{a}_{\overline{20}|} - 20(1+i)^{-20}}{\frac{i^{(2)}}{2}}\right) + 100,000\left(1 + \frac{i^{(2)}}{2}\right)^{-40}$$

$$= 300\left(\frac{\left[\frac{1-(1.0609)^{-20}}{0.0609}(1.0609)\right] - 20(1.0609)^{-20}}{0.03}\right) + 100,000(1.03)^{-40}$$

$$= 59,488.94 + 30,655.68$$

$$= 90,144.62$$

8. A 30 year mortgage loan is being repaid with level monthly payments of 1630.48. The principal in the 90th payment is 352.27.

Determine the **annual effective interest rate** on the loan.

Solution:

$$\text{Principal in }90^{\text{th}}\text{ Payment } = P_{90} = 352.27$$

$$P_k = Qv^{n-k+1}$$

$$n = (30)(12) = 360$$

$$P_{90} = 1630.48v^{360-90+1} = 1630.48v^{271}$$

$$= 1630.48\left(1+\frac{i^{(12)}}{12}\right)^{-271} = 352.27$$

$$\Rightarrow \left(1+\frac{i^{(12)}}{12}\right)^{-271} = \frac{352.27}{1630.48} = 0.216052941$$

$$\Rightarrow \frac{i^{(12)}}{12} = (0.216052941)^{-1/271} - 1 = 0.00567$$

$$\text{Annual Effective Interest Rate } = i$$

$$i = \left(1+\frac{i^{(12)}}{12}\right)^{12} - 1 = (1.00567)^{12} - 1 = 0.070202521 \approx 0.07020$$

9. Brittney is receiving an annuity due with quarterly payments for 17 years. Each quarterly payment is 500 in the first year. Each quarterly payment is 1000 in the second year. Each quarterly payment is 1500 in the third year. The payments continue to increase in the same pattern until each quarterly payment is 8500 in the 17th year.

   Using an interest rate of 8% compounded quarterly, calculate the present value of Brittney's annuity.

   Solution:

   First, we note that the payments are level during each year but increase year to year. Therefore, we know that it is the Formula That Does Not Follow The Rules. Secondly, we note that it is an annuity due.

$$i^{(4)} = 0.08 \Rightarrow \frac{i^{(4)}}{4} = \frac{0.08}{4} = 0.02$$

$$i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.02)^4 - 1 = 0.08243216$$

$$PV = (500)\left(\frac{\ddot{a}_{\overline{17}|} - 17(1+i)^{-17}}{\frac{i^{(4)}}{4}}\right)\left(1 + \frac{i^{(4)}}{4}\right)$$

$$= (500)\left(\frac{\left[\frac{1-(1.08243216)^{-17}}{0.08243216}\right](1.08243216) - 17(1.08243216)^{-17}}{0.02}\right)(1.02)$$

$$= 134{,}976.60$$

*We multiply the present value by (1.02) since it is an annuity due.

10. Ram is receiving a continuous annuity with payments of $13t^2$ at time t for 16 years.

Using a discount function of $v(t) = 1 - 0.03t$, calculate the present value of Ram's annuity.

Solution:

$$PV = \int_0^{16} (13t^2)(1 - 0.03t)dt$$

$$= \int_0^{16} (13t^2 - 0.39t^3)dt$$

$$= \left[\frac{13}{3}t^3 - \frac{0.39}{4}t^4 \right]_0^{16}$$

$$= 11{,}359.57$$

11. A 20 year bond has a maturity value of C and a par value of $0.9C$. The bond pays semi-annual coupons of 250. The bond is bought for a price of 10,367.59 to yield 5% convertible semi-annually.

Determine if this bond is purchased at a discount or a premium and state the amount of discount or premium.

Solution:

$$P = Fra_{\overline{40}|} + Cv^{40}$$

$$10,367.59 = (250)\left(\frac{1-(1.025)^{-40}}{0.025}\right) + (C)(1.025)^{-40}$$

$$10,367.59 = 6275.69 + (0.372430624)(C)$$

$$C = \frac{10,367.59 - 6275.69}{0.372430624} = 10,987.00$$

Since $C > P \Rightarrow Discount = 10,987.00 - 10,367.59 = 619.41$

12. A 20 year bond matures for its par value of 50,000. The bond has semi-annual coupons payable at a rate of 6.2% convertible semi-annually. The bond is bought to yield 7.4% convertible semi-annually.

Calculate the write up of discount (which is synonymous with the amount of principal) in the coupon at the end of the 15th year.

Solution:

To find the write up of discount (amount of principal) in the 30th payment (end of 15th year), we need to find the Book Value (OLB) after the 29th payment.

$$i^{(2)} = 0.074 \Rightarrow \frac{i^{(2)}}{2} = \frac{0.074}{2} = 0.037$$

$$r = \frac{0.062}{2} = 0.031$$

After the 29th payment, there are 11 payments left.

$$\begin{aligned} BV_{29} &= Fra_{\overline{11}|} + Cv^{11} \\ &= (50,000)(0.031)\left(\frac{1-(1.037)^{-11}}{0.037}\right) + (50,000)(1.037)^{-11} \\ &= 47,328.82 \end{aligned}$$

$$I_{30} = (47,328.82)(0.037) = 1751.17$$

$$PMT_{30} = Fr = (50,000)(0.031) = 1550$$

$$P_{30} = PMT_{30} - I_{30} = 1550 - 1751.17 = -201.17$$