MATH 373
Test 4
Spring 2017

May 5, 2017

1. The Bell Life Insurance Company has a two year annuity where it has promised to pay Elizabeth 25,000 at the end of each year for the next two years.

   Bell wants to absolutely match the annuity payments using the following bonds:

   a. Bond A is a one year bond with annual coupons of 100 and a maturity value of 2000.

   b. Bond B is a two year bond with annual coupons of 200 and a maturity value of 1500.

   Determine the number of Bond A that Bell should buy. Assume that you can purchase partial bonds.

2. An annuity due makes annual payments at the beginning of each year for four years.

   Using an interest rate of 7%, calculate the Macaulay Convexity of this annuity.

3. A 20 year bond issued by Talbot Industries has a par value of 10,000. The bond matures for its par value and pays semi-annual coupons at a rate of 8% convertible semi-annually.

   Calculate the Modified Duration of this bond using an annual effective rate of 10.25%.

4. A three year bond pays annual coupons of 500 and has a maturity value of 6000.

   Using an interest rate of 6%, calculate the Modified Convexity of the bond.

5. Using an interest rate of 5%, calculate the Macaulay duration of a perpetuity immediate with payments of 500 at the end of each year.

6. A bond has a price of 100,000 using an interest rate of 6%. The bond also has a modified duration of 22 and a modified convexity of 400 using an interest rate of 6%.

   $E_{MAC}^{\text{1st Order}}$ is the estimated price of the bond using the first order Macaulay approximation if the interest rate changes to 8%.

   $E_{MOD}^{\text{2nd Order}}$ is the estimated price of the bond using the second order Modified approximation if the interest rate changes to 8%.

   Calculate $E_{MAC}^{\text{1st Order}} - E_{MOD}^{\text{2nd Order}}$.

7. The Trout Life Insurance Company owns the following bonds:

    a. Bond A is a zero coupon bond maturing for 100,000 at the end of 6 years. This bond can be purchased to yield an annual effective rate of 10%.

    b. Bond B is a 20 year bond with a Macaulay duration of 12 and a Macaulay convexity of 100. This bond has a price of 90,000. The duration, convexity, and price are calculated at an annual effective interest rate of 10%.

    Determine the Modified Convexity of this portfolio.

8. William must pay Zai 1,000,000 at the end of 12 years.

   William has used Reddington immunization using the following to bonds to protect himself from interest rate changes.

   a. Bond 1 is a zero coupon with a maturity value of 25,000 at the end of X years.

   b. Bond 2 is a zero coupon bond maturing for 15,000 at the end of 19 years.

   Based on an interest rate of 7.5%, William purchased 46.0847 bonds identical to Bond 2.

   Determine X.

9. Lauren purchased a two year bond with semi annual coupons of 200 and a maturity value of 6000. The bond can be purchased to yield 4.2% convertible semi-annually.

The bond was priced based on the following spot interest rate curve:

Time t	**Spot Rate r_t**
0.5	0.0350
1	0.0375
1.5	0.0400
2.0	r_2
2.5	$r_{2.5}$

Determine r_2.

10. Using the following spot interest rate curve, calculate the present value of an annuity due with annual payments of 40,000 for three years:

Time t	**Spot Rate r_t**
1	0.060
2	0.062
3	0.065

11. Lauren can purchase the following three bonds:

    a. Bond 1 is a one year bond with annual coupons of 500 and a maturity value of 4500. The bond sells for 4700.

    b. Bond 2 is a two year bond with annual coupons of 1000 and a maturity value of 5000. This bond has a price of 6180.

    c. Bond 3 is a three year zero coupon bond with a maturity value of 10,000 and a price of 8025.

    Instead, Lauren decides to purchase a three year annuity immediate with payments of 13,000 at the end of each year for three years.

    Determine the present value of Lauren's annuity.

12. You are given the following spot interest rates:

Time t	Spot Rate r_t
1	0.043
2	0.046
3	0.051
4	0.054
5	0.056

Calculate $f_{[2,4]}$.

You are given the following spot interest rates and information for problems 13-14:

Time t	Spot Rate r_t
1	0.043
2	0.046
3	0.051
4	0.054
5	0.056

Katarina has a loan from Hemenway Bank. The loan is for 300,000 the first year. At the end of one year, Katarina repays 100,000 leaving a loan of 200,000 during the second year. At the end of the second year, Katarina repays another 100,000 leaving a loan of 100,000 during the third year.

Katarina will pay Hemenway Bank a variable interest rate equal to the one year spot interest rate at the beginning of each year.

Katarina would like to have a fixed interest rate so she enters into an interest rate swap with Lily. Under the interest rate swap, Katarina will pay a fixed rate to Lily and Lily will pay a variable rate to Katarina. The variable rate will be the same rate that Katarina is paying to Hemenway Bank. The other terms of the swap will mirror the loan that Katarina has.

13. Questions a. through d. are considered one question:

    a. This is an accreting swap.

    True or **False**

    b. What is the settlement period for this swap?

    c. State the notional amount of this swap?

    d. List the counterparties to the swap.

14. Calculate the swap interest rate for Katarina's swap.

15. You are given the following spot interest rates:

Time t	Spot Rate r_t
1	0.043
2	0.046
3	0.051
4	0.054
5	0.056

Tommy purchases a deferred interest rate swap with a term of five years. Under the swap, there is no swapping of interest rates during the first two years. During the last three years, the settlement period will be one year. Under this swap, Tommy will be the payer. The variable interest rate will be based on the one year spot rate at the start of each settlement period.

The notional amount of this swap is 500,000.

Calculate the swap rate for this swap.

16. Miaoqi and Nui entered into a four year interest rate swap on May 5, **2015**. The notional amount of the swap was a level 250,000 for all four years. The swap has annual settlement periods with the first period starting on May 5, **2015**.

    Under the swap, Miaoqi agreed to pay a variable rate based on the one year spot rate at the beginning of each settlement period. Nui will pay Miaoqi the fixed rate of 4% on each settlement date.

    On May 5, **2017**, the spot interest rate curve was as follows:

Time t	**Spot Rate r_t**
1	0.038
2	0.041
3	0.043
4	0.045
5	0.047

    Miaoqi decides that she wants to sell the swap on May 5, **2017**.

    Calculate the market value of the swap on May 5, **2017** from Miaoqi's position in the swap.

17. Beckley Farms has a 500,000 loan from Bailey Bank. Under the terms of the loan, Beckley will pay interest annually to Bailey Bank based on LIBOR plus 120 basis points. Additionally, Beckley will pay the principal of 500,000 at the end of five years.

    Beckley would prefer to know the annual interest cost that will be incurred. To fix the interest rate on the loan, Beckley enters into a five-year interest rate swap with a notional amount of 500,000 and annual settlement dates. The terms of the swap are that Beckley will make swap payments based on a fixed rate of 5.35% and will receive swap payments based on a variable rate of LIBOR plus 50 basis points.

    During the third year of the loan, LIBOR is 5.6%.

    Determine the net **interest** payment made by Beckley at the end of the third year.

18. The current spot interest rate curve is as follows:

t	r_t		t	r_t
0.25	1.50%		1.75	2.40%
0.50	1.65%		2.00	2.48%
0.75	1.79%		2.25	2.80%
1.00	1.92%		2.50	3.10%
1.25	2.10%		2.75	3.35%
1.50	2.25%		3.00	3.50%

Rafael has a one year loan for 1,000,000 which has a variable interest rate that resets at the beginning of each three month period. The interest rate will be the spot interest rate at the beginning of each three month period.

Rafael enters into an interest rate swap where he is the payer with the characteristics of the swap exactly match the loan.

Determine the quarterly swap rate that Rafael will pay.

19. You are given the following zero coupon bond prices for a zero coupon bond that matures for 1 on the maturity date:

Maturity Date	Price
1 Year	0.965
2 Years	0.920
3 Years	0.875
4 Years	0.825
5 Years	0.770

Josh and Phillip enter into a four year swap with a notional amount of 200,000. The swap has annual settlement periods. Under the swap, Josh will pay Phillip the fixed swap rate at the end of each year while Phillip will pay Josh the variable rate where the variable rate is the one year spot rate at the beginning of each year.

Determine the net swap payment at the end of the first year. Be sure to state who makes the payment and who receives the payment.