MATH 373
Quiz 1
Spring 2018

January 30, 2018

1. Alisa invests 42,000 in Amir Bank. At the end of 10 years, Alisa has 100,000.

   Amir Bank pays interest based on the following:

   a. The first two years, Amir pays an annual effective interest rate of i .

   b. During the next three years, Amir pays a nominal discount rate of 8% compounded quarterly.

   c. During the last five years, Amir pays a force of interest equal to $\delta_t = 0.04 + 0.001t^2$ where t is measured from the date of the original investment of 42,000.

   Determine i .

   Solution:

$$(42,000)(1+i)^2\left(1-\frac{0.08}{4}\right)^{-4(3)} e^{\int_5^{10}\left(0.04+0.001r^2\right)dr} = 100,000$$

$$\boxed{e^{\int_5^{10}\left(0.04+0.001r^2\right)dr} = e^{\left[0.04r+\frac{0.001}{3}r^3\right]_5^{10}} = e^{(0.4+0.333333333-0.2-0.0411666666)} = e^{0.49166666}}$$

$$(1+i)^2 = \frac{100,000}{(42,000)\left(1-\frac{0.08}{4}\right)^{-4(3)} e^{0.49166666}} = 1.14708605$$

$$i = (1.14708605)^{0.5} - 1 = 0.068975493$$

2. You are given that $v(t) = \left[1+\beta t^2\right]^{-1}$.

   You are also given that $\delta_5 = \delta_{10}$.

   Determine β .

Solution:

$$v(t) = \frac{1}{1+\beta t^2} = \frac{1}{a(t)} ==> a(t) = 1+\beta t^2$$

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{2\beta t}{1+\beta t^2}$$

$$\delta_5 = \delta_{10} ==> \frac{2\beta(5)}{1+\beta(5)^2} = \frac{2\beta(10)}{1+\beta(10)^2} ==> 10\beta\left[1+100\beta\right] = 20\beta\left[1+25\beta\right]$$

$$==> \left[1+100\beta\right] = 2\left[1+25\beta\right] ==> 1+100\beta = 2+50\beta$$

$$50\beta = 1 ==> \beta = 0.02$$

3. Jordyn invests 1000 in Bank Chen. Bank Chen pays simple interest rate of s. In the 10^{th} year, Jordyn earns an annual effective interest rate of 5%.

   Calculate the amount of money Jordyn will have at the end of the 10^{th} year.

   Solution:

   By Definition

   $$\text{The effective interest rate in the nth year} = i_n = \frac{a(n)-a(n-1)}{a(n-1)}$$

   Which for simple interest is

   $$\frac{a(n)-a(n-1)}{a(n-1)} = \frac{\left(1+(n)s\right)-\left(1+(n-1)s\right)}{1+(n-1)s} = \frac{1+ns-1-ns+s}{1+(n-1)s} = \frac{s}{1+(n-1)s}$$

   $$i_{10} = 0.05 = \frac{s}{1+(10-1)(s)} = \frac{s}{1+9s} \text{ ==> } 0.05(1+9s) = s$$

   $$\text{==> } 0.05+0.45s = s \text{ ==> } 0.05 = 0.55s \text{ ==> } s = \frac{1}{11}$$

   $$A(10) = (1000)\left[1+\left(\frac{1}{11}\right)(10)\right] = 1909.09$$