MATH 373
Quiz 4
Spring 2018
March 8, 2018

1. Caroline has the option of the following two continuous annuities:

    a. An annuity that pays continuously at an annual rate of 1000 for 20 years.

    b. An annuity that pays continuously at a rate of Xt at time t for 20 years.

   Both annuities have the same present value at a force of interest of 8%.

   Determine X.

   Solution:

   $Option\ a = Option\ b$

   $$1000\bar{a}_{\overline{20|}} = X\left[(\bar{I}\bar{a})_{\overline{20|}}\right]$$

   $$1000\left(\frac{1-e^{-0.08(20)}}{0.08}\right) = X\left[\frac{\dfrac{1-e^{-0.08(20)}}{0.08} - 20e^{-0.08(20)}}{0.08}\right]$$

   $$9976.29 = X(74.2295) \Longrightarrow X = 134.40$$

2. Kevin is receiving an annuity due with quarterly payments for the next 30 years. Each payment in the first year is P. Each payment in the second year is $2P$. Each payment in the third year is $3P$. The payments continue to increase by P each year until each payment in the 30th year is $30P$.

The present value of this annuity at an interest rate of 8% compounded quarterly is 200,000.

Determine P.

Solution:

Since this the formula that does not follow the rules, we need both $\dfrac{i^{(4)}}{4}$ and i.

We are given that $i^{(4)} = 0.08 ==> \dfrac{i^{(4)}}{4} = 0.02$ and $i = (1.02)^4 - 1 = 0.08243216$.

$$200,000 = P \left[\frac{\ddot{a}_{\overline{30}|0.08243216} - 30(1.08243216)^{-30}}{0.02} \right] (1.02) =$$

$$P \left[\frac{\left(\dfrac{1-(1.08243216)^{-30}}{0.08243216} \right)(1.08243216) - 30(1.08243216)^{-30}}{0.02} \right] (1.02) =$$

$$P \left[\frac{11.91140233 - 2.786866898}{0.02} \right] (1.02) = P(465.356407)$$

$$P = \frac{200,000}{465.356407} = 429.78$$

The multiplication by (1.02) at the end of the formula is because this is "an annuity due".

3. I would like to receive 10 points for having written my name on the front cover. Circle the correct answer.

True   or   **False**