MATH 373
Quiz 6
Spring 2018

April 12, 2018

1. An annuity due pays 20,000 at the beginning of each year for 20 years.

   Calculate the Modified Duration of this annuity at an annual effective interest rate of 8%.

   Solution:

$$ModDur = v\frac{\sum C_t(t)v^t}{\sum C_t v^t}$$

$$= (1.08)^{-1}\frac{(20,000)(0)(1.08)^0 + (20,000)(1)(1.08)^1 + \ldots + (20,000)(19)(1.08)^{19}}{(20,000)(1.08)^0 + (20,000)(1.08)^1 + \ldots + (20,000)(1.08)^{19}}$$

$$= (1.08)^{-1}\frac{(20,000)(1)(1.08)^1 + \ldots + (20,000)(19)(1.08)^{19}}{(20,000)(1.08)^0 + (20,000)(1.08)^1 + \ldots + (20,000)(1.08)^{19}}$$

$$= (1.08)^{-1}\frac{(20,000)\left(\frac{1-(1.08)^{-19}}{0.08}\right) + \left(\frac{20,000}{0.08}\right)\left(\frac{1-(1.08)^{-19}}{0.08} - 19(1.08)^{-19}\right)}{(20,000)\left(\frac{1-(1.08)^{-20}}{0.08}\right)(1.08)}$$

$$= (1.08)^{-1}\frac{1,492,339.48}{212,071.984} = 6.5157$$

2. A three year bond has a maturity value of 10,000 and annual coupons of 600.

   Calculate the Modified Convexity of this bond at an annual effective interest rate of 6%.

   Solution:

$$ModCon = v^2 \frac{\sum C_t (t)(t+1) v^t}{\sum C_t v^t}$$

$$= (1.06)^{-2} \frac{(600)(1)(2)(1.06)^{-1} + (600)(2)(3)(1.06)^{-2} + (10,600)(3)(4)(1.06)^{-3}}{(600)(1.06)^{-1} + (600)(1.06)^{-2} + (10,600)(1.06)^{-3}}$$

$$= 9.8910$$

3. A ten year bond sold by Maples & Mills LTD has a price of 100,000. The Macaulay Duration of the bond is 6 and the Macaulay Convexity is 34. The price, the Macaulay Duration, and the Macaulay Convexity are all calculated using an annual effective interest rate of 7%.

Using the second modified approximation, estimate the price of this bond at an annual effective interest rate of 6%.

Solution:

$$P(i) = P(i_0)\left[1-(i-i_0)(ModDur)+\frac{(i-i_0)^2}{2}(ModCon)\right]$$

$$i_0 = 0.07 \quad \text{and} \quad i = 0.06$$

$$ModDur = v(MacDur) = (1.07)^{-1}(6) = 5.607476636$$

$$ModCon = v^2(MacDur + MacCon) = (1.07)^{-2}(6+34) = 34.93754913$$

$$P(i) = (100,000)\left[1-(0.07-0.06)(5.607476636)+\frac{(0.07-0.06)^2}{2}(34.93754913)\right]$$

$$=105,782.16$$