MATH 373
Quiz 7
Spring 2018
April 19, 2018

1. Allison Kelly & Company must pay 300,000 to Emily at the end of 13 years. The Company wants to use Reddington Immunization to protect itself from interest rate changes. The company will use the following two bonds to immunize itself at an annual effective interest rate of 5%:

    a. Bond 1 is a zero coupon bond that matures for 10,000 at the end of 8 years.

    b. Bond 2 is a zero coupon bond that matures for 10,000 at the end of 20 years.

    Assuming Allison Kelly can buy any number of the above bonds including partial bonds, determine the number of Bond 2 that should be purchased.

    Solution:

    First we must find the amount spent on each bond and then calculate the amount of each bond.

$$\text{Amount Spend on Bond 2} = \frac{D^L - D^1}{D^2 - D^1}\left[PV \text{ of Liability}\right]$$

$$= \frac{13-8}{20-8}\left[300,000(1.05)^{-13}\right] = 66,290.17$$

$$\text{Price of Bond 2} = (10,000)(1.05)^{-20} = 3768.89483$$

$$\text{Number of Bond 2} = \frac{\text{Amount Spent}}{\text{Price}} = \frac{66,290.17}{3768.89483} = 17.59$$

2. You are given the following spot interest rate curve:

t	r_t
0.5	1.25%
1.0	2.00%
1.5	2.75%
2.0	3.20%
2.5	3.60%
3.0	4.00%

Using these spot interest rates, determine the price of a 2 year bond that matures for 100,000 and has semi-annual coupons of 5000.

Solutions:

$$PV = 5000(1.0125)^{-0.5} + 5000(1.02)^{-1} + 5000(1.0275)^{-1.5} + 105,000(1.032)^{-2}$$

$$= 113,260.95$$

3. You are given the following three bonds:

   a. A one year bond with annual coupons of 100 and a maturity value of 1000. The bond has a price of 1050.

   b. A two year bond with annual coupons of 80 and maturity value of 1000. The bond has a price of 990.

   c. A three year bond with annual coupons of 200 and a maturity value of 800. The price of this bond is 1065.

   Use bootstrapping to determine the three year spot interest rate.

   Solution:

   Using one year bond:

$$1050 = 1100(1+r_1)^{-1} \Longrightarrow r_1 = \frac{1100}{1050} - 1 = 0.047619$$

   Using two year bond:

$$990 = 80(1+r_1)^{-1} + 1080(1+r_2)^{-2} = 80(1.047619)^{-1} + 1080(1+r_2)^{-2}$$

$$\Longrightarrow r_2 = \left(\frac{1080}{990 - 80(1.047619)^{-1}} \right)^{0.5} - 1 = 0.087239$$

   Using three year bond:

$$1065 = 200(1+r_1)^{-1} + 200(1+r_2)^{-2} + 1000(1+r_3)^{-3} =$$

$$200(1.047619)^{-1} + 200(1.087239)^{-2} + 1000(1+r_3)^{-3}$$

$$\Longrightarrow r_3 = \left(\frac{1000}{1065 - 200(1.047619)^{-1} - 200(1.087239)^{-2}} \right)^{1/3} - 1 = 0.123633$$