Math 373
Test 3
Spring 2018
May 2, 2018

1. Saqqa Corporation pays quarterly dividends with the next dividend of 4.00 being paid in one month. Saqqa is a rapidly growing company and expects each dividend to increase by 3% over the previous dividend. In other words, the dividend paid in one month will be 4.00, the dividend paid in four months will be $4.00(1.03)$, the dividend to be paid in seven months will be $4.00(1.03)^2$, etc.

   Using the dividend discount method, determine the price of a share of Saqqa stock using an interest rate of 20% compounded quarterly.

   Solution:

   We need $\frac{i^{(4)}}{4}$ which is $\frac{0.20}{4} = 0.05.$

   $$PV = (4.00v + 4.00(1.03)v^2 + ...)(1.05)^{2/3}$$

   $$= \frac{4(1.03)^{-1} - 0}{1 - (1.03)(1.05)^{-1}}(1.05)^{2/3} = 206.61$$

2. The stock of White Industries pays quarterly dividends. The next dividend is payable in three months and will be 5.00. The dividends for White Industries are expected to increase with each dividend being 0.10 larger than the previous dividend. In other words, the first dividend will be 5.00. The second dividend will be 5.10. The third dividend will be 5.20, etc.

   Alisa purchases the stock for 200.

   Using the dividend discount method, determine the annual effective interest rate that Alisa expects to earn on the stock.

   Solutions:

   $$PV = 200 = \frac{5}{i} + \frac{0.1}{i^2} \text{ ==> } 200i^2 - 5i - 0.1 = 0$$

   $$i = \frac{5 \pm \sqrt{(-5)^2 - 4(200)(-0.10)}}{2(200)} = 0.038117377$$

   Since payments are quarterly, $\frac{i^{(4)}}{4} = 0.038117377$

   $$i = (1.038117377)^4 - 1 = 0.1614108$$

3. The Bowman Insurance Company will pay Katie 250,000 at the end of each year for the next four years. Bowman wants to exactly match the payments using the following four bonds:

    a. Bond 1 is a one year bond with annual coupons of 400 and a maturity value of 10,000.

    b. Bond 2 is a two year bond with annual coupons of 500 and a maturity value of 5000.

    c. Bond 3 is a three year bond with annual coupons of 200 and a maturity value of 2300.

    d. Bond 4 is a zero coupon bond maturing in four years with a maturity value of 20,000.

    Determine the number of Bond 1 which Bowman should purchase.

    Solution:

$$\text{Time 1==>}(Bond1)(10,400)+(Bond2)(500)+(Bond3)(200)+(Bond4)(0)=250,000$$
$$\text{Time 2==>}(Bond1)(0)+(Bond2)(5500)+(Bond3)(200)+(Bond4)(0)=250,000$$
$$\text{Time 3==>}(Bond1)(0)+(Bond2)(0)+(Bond3)(2500)+(Bond4)(0)=250,000$$
$$\text{Time 4==>}(Bond1)(0)+(Bond2)(0)+(Bond3)(0)+(Bond4)(20,000)=250,000$$

$$\text{Using Time 3==>}Bond3=\frac{250,000}{2500}=100$$

$$\text{Using Time 2==>}Bond2=\frac{250,000-(100)(200)}{5500}=41.81818$$

$$\text{Using Time 1==>}Bond1=\frac{250,000-(100)(200)-(41.81818)(500)}{10,400}=20.1049$$

4. Ashley just got fired by Huljack LTD. Huljack has agreed to make the following payments to Ashley as a severance package:

   a. Payment of 100,000 today;

   b. Payment of 200,000 at the end of two years; and

   c. Payment of 400,000 at the end of four years.

   Calculate the Modified Convexity of Ashley's payments at an annual effective interest rate of 8%.

   Solution:

$$ModCon = v^2 \frac{\sum C_t (t)(t+1)v^t}{\sum C_t v^t} =$$

$$(1.08)^{-2} \frac{(200,000)(2)(3)(1.08)^{-2} + (400,000)(4)(5)(1.08)^{-4}}{100,000 + (200,000)(1.08)^{-2} + (400,000)(1.08)^{-4}} = 10.475$$

5. A 25 year bond pays semi-annual coupons of 200 and matures for 1000.

Calculate the modified duration of this bond at an annual effective interest rate of 12.36%.

Solution:

$$MacDur = \frac{\sum C_t(t)v^t}{\sum C_t v^t}$$

$$= \frac{200(0.5)v^{0.5} + 200(1)v + 200(1.5)v^{1.5} + ... + (200)(25)v^{25} + (1000)(25)v^{25}}{200a_{\overline{50|}0.06} + 1000(1.06)^{-50}}$$

$$= \frac{100v^{0.5} + 200v^{1} + 300v^{1.5} + ... + 5000v^{25} + (25,000)v^{25}}{200a_{\overline{50|}0.06} + 1000(1.06)^{-50}}$$

$$= \frac{100(1.06)^{-1} + 200(1.06)^{-2} + 300(1.06)^{-3} + ... + 5000(1.06)^{-50} + (25,000)(1.1236)^{-25}}{200a_{\overline{50|}0.06} + 1000(1.06)^{-50}}$$

$$= \frac{100\left(\frac{1-(1.06)^{-50}}{0.06}\right) + \left(\frac{100}{0.06}\right)\left(\frac{1-(1.06)^{-50}}{0.06} - 50(1.06)^{-50}\right) + (25,000)(1.1236)^{-25}}{200a_{\overline{50|}0.06} + 1000(1.06)^{-50}}$$

$$= 7.69621$$

$$ModDur = v(MacDur) = (1.1236)^{-1}(7.69621) = 6.8496$$

6. Kevin owns the following bonds. All values are calculated using an annual effective interest rate of 5%.

	Macaulay Duration	Macaulay Convexity	Price
Bond A	10	90	30,000
Bond B	16	200	45,000
Bond C	8	55	25,000

Determine the modified convexity of Kevin's bond portfolio at an annual effective interest rate of 5%

Solution:

$$ModCon \text{ for Bond A} = (10+90)v^2 = 100v^2$$

$$ModCon \text{ for Bond B} = (16+200)v^2 = 216v^2$$

$$ModCon \text{ for Bond C} = (8+55)v^2 = 63v^2$$

$$C_{ModCon}^{Port} = \frac{\sum P^t \cdot C^t}{\sum P^t} = \frac{(30,000)(100)(1.05)^{-2} + (45,000)(216)(1.05)^{-2} + (25,000)(63)(1.05)^{-2}}{30,000 + 45,000 + 25,000}$$

$$= 129.66$$

7. Beau is receiving the following payments from a trust fund:

    a. 200,000 at time 2

    b. 400,000 at time 10

    c. 800,000 at time 16

    Calculate the Macaulay Convexity of these payments at an interest rate of 7%.

    Solution:

$$MacCon = \frac{\sum C_t (t^2) v^t}{\sum C_t v^t}$$

$$= \frac{(200,000)(2^2)(1.07)^{-2} + (400,000)(10^2)(1.07)^{-10} + (800,000)(16^2)(1.07)^{-16}}{(200,000)(1.07)^{-2} + (400,000)(1.07)^{-10} + (800,000)(1.07)^{-16}}$$

$$= 139.80$$

8. A 10 year bond has a price of 24,000. The Macaulay duration of the bond is 12. The Modified convexity of the bond is 123. All values are calculated using an annual effective interest rate of 4.5%.

$P_1 =$ the estimated price of this bond using the first order Macaulay approximation at an annual effective interest rate of 5.75%.

$P_2 =$ the estimated price of this bond using the first order Modified approximation at an annual effective interest rate of 5.75%.

Determine $P_1 - P_2$.

Solution:

$$ModDur = MacDur(1.045)^{-1} = (12)(1.045)^{-1} = 11.4832536$$

$$P_1 = (24,000)\left[\frac{1.045}{1.0575}\right]^{12} = 20,808.57$$

$$P_2 = (24,000)(1-(0.0575-0.045)(11.4832536)) = 20,555.02$$

$$P_1 - P_2 = 20,808.57 - 20,555.02 = 253.55$$

9. Maxwell Company agrees to pay Chloe 850,000 at the end of 10 years. Chrissy, who is the Chief Actuary for Maxwell Company, wants to use Full Immunization with the following two bonds:

    a. Bond A is a zero coupon bond which matures for 10,000 at the end of 6 years.

    b. Bond B is a zero coupon bond which matures for 12,000 at the end of 13 years.

   Maxwell Company can purchase any number of each bond including partial bonds.

   At an annual effective interest rate of 9%, determine the number of each bond that Maxwell Company should purchase.

   Solution:

$$AmountBondA = \frac{D^B - D^L}{D^B - D^A}(PV\ \ of\ \ Liability) = \left(\frac{13-10}{13-6}\right)(850,000)(1.09)^{-10} = 153,878.22$$

$$NumberBondA = \frac{AmountBondA}{\Pr iceBondA} = \frac{153,878.22}{(10,000)(1.09)^{-6}} = 25.807$$

$$AmountBondB = \frac{D^L - D^A}{D^B - D^A}(PV\ \ of\ \ Liability) = \left(\frac{10-6}{13-6}\right)(850,000)(1.09)^{-10} = 205,170.96$$

$$NumberBondB = \frac{AmountBondB}{\Pr iceBondB} = \frac{205,170.96}{(12,000)(1.09)^{-13}} = 52.418$$

10. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	3.00%	3.0	4.85%
1.0	3.50%	3.5	5.20%
1.5	3.95%	4.0	5.35%
2.0	4.30%	4.5	5.45%
2.5	4.60%	5.0	5.50%

Determine the present value of an annuity immediate with semi-annual payments for two years. The payments increase each payment. The first payment is 100. The second payment is 200. The third payment is 400. The final payment is 800.

Solution:

$$PV = 100(1.03)^{-0.5} + 200(1.035)^{-1} + 400(1.0395)^{-1.5} + 800(1.043)^{-2}$$

$$= 1404.58$$

11. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	3.00%	3.0	4.85%
1.0	3.50%	3.5	5.20%
1.5	3.95%	4.0	5.35%
2.0	4.30%	4.5	5.45%
2.5	4.60%	5.0	5.50%

Determine the accumulated value of an annuity due that pays 1000 at the beginning of each year for 3 years.

Solution:

First, we find the present value and then find the accumulated value.

$$PV = 1000(1 + (1.035)^{-1} + (1.043)^{-2}) = 2885.43$$

$$AV = PV(1.0485)^{3} = 3325.95$$

12. You are given the following two bonds:

    a. Bond 1 is a one year zero coupon bond with a price of 9400 and a maturity value of 10,000.

    b. Bond 2 is a two year bond with annual coupons of 300 and a maturity value of 2000. This bond sells for an annual yield of 8%.

    You are also given that the three year spot interest rate is 9%.

    Determine the price of a three year bond with annual coupons of 800 and a maturity value of 3000.

    Solution:

    Using Bond 1

$$9400 = \frac{10,000}{1+r_1} \text{ ==> } r_1 = \frac{10,000}{9400} - 1 = 0.063829787$$

    Using Bond 2

$$\frac{300}{1+r_1} + \frac{2300}{(1+r_2)^2} = \frac{300}{1.08} + \frac{2300}{(1.08)^2} \text{ ==> } \frac{300}{1.063829787} + \frac{2300}{(1+r_2)^2} = 2249.66$$

$$r_2 = \frac{2300}{2249.66 - \frac{300}{1.063829787}} - 1 = 0.081158118$$

$$\text{Price} = \frac{800}{1+r_1} + \frac{800}{(1+r_2)^2} + \frac{3800}{(1+r_3)^3} = \frac{800}{1.063829787} + \frac{800}{(1.081158118)^2} + \frac{3800}{(1.09)^3} = 4370.70$$

You are given the following spot interest rates and information for questions 13 and 14:

t	r_t	t	r_t
0.5	3.00%	3.0	4.85%
1.0	3.50%	3.5	5.20%
1.5	3.95%	4.0	5.35%
2.0	4.30%	4.5	5.45%
2.5	4.60%	5.0	5.50%

Chen Corporation has a three year loan from Luo Bank for an amount of 500,000. The loan has a variable interest rate which is equal to the one year spot rate at the beginning of each year of the loan. Chen will pay the interest on the loan at the end of each year for three years. Additionally, Chen will pay the principal of the loan which is 500,000 at the end of the third year.

Chen would like to have a fixed interest rate instead of a variable interest rate. Therefore, Chen enters into an interest rate swap with Zhu Investment Bank. Under the swap, Chen will pay a fixed interest rate to Zhu and Zhu will pay the variable interest rate to Chen. The terms of the swap mirror the terms of the loan.

13. Answer the following three questions which will count as one question for this exam.

    a. State the counterparties to the interest rate swap.

       Chen is the payer and Zhu is the receiver.

    b. State the tenor of the swap.

       Three Years

    c. State the notional amount of the swap.

       500,000

14. Determine the fixed interest rate under this swap.

Solution:

$$R = \frac{1-P_3}{P_1+P_2+P_3} = \frac{1-(1.0485)^{-3}}{(1.035)^{-1}+(1.043)^{-2}+(1.0485)^{-3}} = 0.04811$$

You are given the following information for Questions 15 and 16.

Sadi and Yash enter into a five year interest rate swap with annual swap payments. Under the swap, Sadi will pay a fixed interest rate of 6.4% to Yash and Yash will pay the variable interest rate to Sadi. The variable interest rate will be the one year spot interest rate at the beginning of each year.

The notional amount of the swap is 100,000.

At the end of three years, there are two years left on the swap. The spot interest rates at the end of three years are:

t	r_t
1	0.060
2	0.067
3	0.072
4	0.075
5	0.077

15. Determine the net swap payment in the fourth year of the swap. State who pays this payment.

Solution:

Sadi owes (100,000)(0.064)=6400

Yash owes (100,000)(0.060)=6000

Sadi Pays Yash 400

16. If at the end of the third year Sadi decides to sell the swap to Daniel, determine the market value of the swap (from Sadi's viewpoint) at the end of three years.

Solution:

$$\text{Market Value} = PV \;\; of \;\; \text{Pr}\,ojected \;\; Cashflows$$

$$f_{[1,2]} = \frac{(1.067)^2}{1.06} - 1 = 0.074046226$$

$$= \frac{100,000(0.06 - 0.064)}{1.06} + \frac{100,000(0.074046226 - 0.064)}{(1.067)^2} = 505.06$$

17. You are given the following spot interest rates:

t	r_t
1.0	3.50%
2.0	4.30%
3.0	4.85%
4.0	5.35%
5.0	5.50%

Andrew and Michael enter into a three year interest rate swap. This is a deferred swap as there are no swap payments at the end of the first year. Annual swap payments will be made at the end of the last two years. Andrew will be the payer and Michael will be the receiver under the swap. The variable interest rate will be the one year spot interest rate at the beginning of each year.

The notional amount of the swap during the second year is 200,000. The notional amount of the swap during the third year is 400,000.

Determine the fixed interest rate under this swap and state whether Andrew or Michael will be paying this interest rate.

Solutions:

Andrew pays the fixed rate.

$$f_{[1,2]} = \frac{(1.043)^2}{1.035} - 1 = 0.051061836$$

$$f_{[2,3]} = \frac{(1.0485)^3}{(1.043)^2} - 1 = 0.059587162$$

$$R = \frac{200,000\, f_{[1,2]}(1+r_2)^{-2} + 400,000\, f_{[2,3]}(1+r_3)^{-3}}{200,000(1+r_2)^{-2} + 400,000(1+r_3)^{-3}}$$

$$= \frac{200,000(0.051061836)(1.043)^{-2} + 400,000(0.059587162)(1.0485)^{-3}}{200,000(1.043)^{-2} + 400,000(1.0485)^{-3}}$$

$$= 0.056635$$

18. NULL INC has a 1,000,000 loan from Zajac Bank. Under the terms of the loan, NULL will pay interest annually to Zajac Bank based on LIBOR plus 120 basis points. Additionally, NULL will pay the principal of 1,000,000 at the end of ten years.

NULL would prefer to know the annual interest cost that will be incurred. To fix the interest rate on the loan, NULL enters into a ten-year interest rate swap with a notional amount of 1,000,000 and annual settlement dates. The terms of the swap are that NULL will swap a variable rate of LIBOR plus 70 basis points for a fixed rate of 6.2%%.

During the third year of the loan, LIBOR is 5.7%.

Determine the net interest payment that NULL will make during the third year.

Solution:

Payment to the Bank

$$1,000,000(0.057+0.012)=69,000$$

Payment from the Swap

$$1,000,000(0.057+0.007-0.062)=2000$$

$$\text{Net Interest Payment}=69,000-2000=67,000$$