Math 373
Spring 2019
Quiz 3
March 7, 2019

1. Nathan has won the lottery! The lottery will make annual payments at the beginning of each year for the next 25 years. The first payment is 100,000. The second payment is 150,000. The third payment is 200,000. The same pattern continues with each payment being 50,000 more than the prior payment until a payment of 1,300,000 is made at the start of the 25th year.

   Calculate the present value of these payments at an annual effective interest rate of 9%.

   Solution:

   This is the P&Q Formula. We note that it is an annuity due.

$$PV = \left[100{,}000 a_{\overline{25|}} + \frac{50{,}000}{0.09}\left(a_{\overline{25|}} - 25(1.09)^{-25}\right)\right](1.09)$$

$$a_{\overline{25|}} \frac{1-(1.09)^{-25}}{0.09} = 9.822579605$$

$$PV = 5{,}263{,}154.65$$

2. Kate invests 2500 into the Adams Fund at the end of each year for the next 13 years. The Adams Fund earns an annual effective interest rate of 7%.

   At the end of each year, the interest earned in the Adams Fund is transferred to the Baker Fund. The Baker fund earns an annual effective interest rate of 8%.

   Determine the amount that Kate will have at the end of 13 years when she combines the amount in the Adams Fund and the amount in the Baker Fund.

Solution:

There will be 13 payments of 2500 deposited into the Adams Fund. Since the interest from the Adams Fund is withdrawn each year, the amount in the Adams Fund at the end of 13 years is just the money deposited which will be (13)(2500) = 32,500.

Each year, the interest earned by the Adams Fund will be transfered to the Baker Fund. The amount transfered to Baker Fund is 175 at the of year 2, 350 at the end of year 3, etc. Note that there is no transfer at the end of year 1 because there was no money in Adams Fund during the first year. The Baker Fund earns 8% so the amount in the Baker Fund at end of 13 years is:

$$AV = \left[175a_{\overline{12|}} + \frac{175}{0.08}(a_{\overline{12|}} - 12(1.08)^{-12}\right](1.08)^{12} = 18,583.46$$

$$a_{\overline{12|}} = \frac{1-(1.08)^{-12}}{0.08} = 7.536078017$$

$$Total = 32,500 + 18,583.46 = 51,083.46$$

3. The Crawford Family Trust fund will pay Emily monthly payments at the end of each month for 5 years. The first payment is 1000. Each payment after that payment is 102% of the prior payment.

   In other words, the first payment will be 1000. The second payment will be $1000(1.02)^1$. The third payment will be $1000(1.02)^2$, etc.

   Calculate the present value of these payments at an interest rate of 9% compounded monthly.

   Solution:

   Payments are 1000, 1000(1.02), $1000(1.02)^2$, ..., $1000(1.02)^{59}$

$$PV = 1000v + 1000(1.02)v^2 + \ldots + (1000)(1.02)^{59}v^{60}$$

$$\boxed{\text{We need } \frac{i^{(12)}}{12} \text{ which is equal to } \frac{0.09}{12} = 0.0075.}$$

$$PV = PV = 1000(1.0075)^{-1} + 1000(1.02)(1.0075)^{-2} + \ldots + (1000)(1.02)^{59}(1.0075)^{-60}$$

$$= \frac{1000(1.0075)^{-1} - (1000)(1.02)^{60}(1.0075)^{-61}}{1-(1.02)(1.0075)} = 87,647.47$$