MATH 373
Quiz 6
Spring 2019
April 16, 2019

1. The Filza Insurance Company owes Mary 10,000 at the end of one year and 30,000 at the end of two years. Filza uses exact matching to protect from interest rate changes.

   Filza uses the following two bonds to exactly match the cash flows:

   a. Bond A is a one year bond with annual coupons of 100 and a maturity value of 1600.

   b. Bond B is a two year bond with annual coupons of 60 and a maturity value of 1000.

   Determine the number of each bond that Filza should purchase.

   Solution:

	Time 1	**Time 2**
Payments to Mary	10,000	30,000
Bond A	1700	
Bond B	60	1060

$$30,000 = 1060B \Longrightarrow B = \frac{30,000}{1060} = 28.30189$$

$$10,000 = 60B + 1700A = 60(28.30189) + 1700A$$

$$\Longrightarrow A = \frac{10,000 - (60)(28.30189)}{1700} = 4.88346$$

2. Shyam is receiving an annuity due with 12 annual payments of 200.

   Calculate the Macaulay Duration of Shyam's annuity using an annual effective interest rate of 8%.

   Solution:

$$MacDur = \frac{\sum C_t(t)v^t}{\sum C_t v^t} = \frac{(200)(0)v^0 + (200)(1)v^1 + ... + (200)(11)v^{11}}{200\ddot{a}_{\overline{12}|}}$$

$$\frac{(200)v^1 + ... + (2200)(11)v^{11}}{200\ddot{a}_{\overline{12}|}} = \frac{200a_{\overline{11}|} + \frac{200}{0.08}(a_{\overline{11}|} - 11(1.08)^{-11})}{200\ddot{a}_{\overline{12}|}} =$$

$$\frac{200\left(\frac{1-(1.08)^{-11}}{0.08}\right) + \frac{200}{0.08}\left(\frac{1-(1.08)^{-11}}{0.08} - 11(1.08)^{-11}\right)}{200\left(\frac{1-(1.08)^{-12}}{0.08}\right)(1.08)} = 4.5957$$

3. Alex owns a 15 year bond with annual coupons of 100 and a maturity value of 2000.

   Calculate the Modified Duration of her bond using an annual effective interest rate of 5%.

   Solution:

$$ModDur = v\frac{\sum C_t(t)v^t}{\sum C_t v^t} = v\left(\frac{(100)(1)v^1 + (100)(2)v^2 + \ldots + (100)(15)v^{15} + (2000)(15)v^{15}}{100a_{\overline{15}|} + 2000v^{15}}\right)$$

$$v\left(\frac{100a_{\overline{15}|} + \frac{100}{0.05}(a_{\overline{15}|} - 15(1.05)^{-15}) + (2000)(15)(1.05)^{-15}}{100a_{\overline{15}|} + 2000v^{15}}\right) =$$

$$(1.05)^{-1}\left[\frac{100\left(\frac{1-(1.05)^{-15}}{0.05}\right) + \frac{100}{0.05}\left(\frac{1-(1.05)^{-15}}{0.05} - 15(1.05)^{-15}\right)}{100\left(\frac{1-(1.05)^{-15}}{0.05}\right) + 2000(1.05)^{-15}}\right] = 10.38$$