Math 373
Test 1
Spring 2019
February 26, 2019

1. Kiran wants to have 1,000,000 when she turns 65. Today is Kiran's 25th birthday. In order to accomplish her goal, she will deposit D into an account earning an annual effective interest rate of 8% at the beginning of each year for the next 40 years.

   Determine D.

   Solution:

   $$D\ddot{s}_{\overline{40|}} = 1{,}000{,}000$$

   $$D = \frac{1{,}000{,}000}{\left(\frac{(1.08)^{40}-1}{0.08}\right)(1.08)} = 3574.22$$

   Or

   Set BGN

   $\boxed{N} \leftarrow 40$

   $\boxed{I/Y} \leftarrow 8$

   $\boxed{FV} \leftarrow 1{,}000{,}000$

   $\boxed{CPT}\boxed{PMT} \Rightarrow 3574.22$

2. Kaitlyn invests 10,000 in an account earning simple interest. At the end of 10 years, Kaitlyn has 20,000.

   Jalen invests 10,000 in an account earning compound interest.

   The effective interest rate during the tenth year for Kaitlyn is equal to the effective interest rate during the tenth year for Jalen.

   Determine the amount that Jalen has in his account at the end of 10 years.

   Solution:

   Kaitlyn

$$10,000(1+10s) = 20,000 ==> 1+10s = 2 ==> 10s = 1 ==> s = 0.1$$

$$i_{10}^{Kaitlyn} = i_{10}^{Jalen}$$

$$i_{10}^{Kaitlyn} = \frac{s}{1+(n-1)s} = \frac{0.1}{1+9(0.1)} = 0.052631579$$

$$i_{10}^{Jalen} = i = 0.052631579$$

$$Amount = (10,000)(1+0.052631579)^{10} = 16,701.83$$

3. You are given:

   a. $v(t) = \dfrac{1}{\alpha + \beta t^2}$

   b. $\delta_5 = 0.08$

   Peyton invests X today in an account and has 25,000 at the end of 10 years.

   Determine X.

   Solution:

$$v(t) = \frac{1}{a(t)} ==> a(t) = \alpha + \beta t^2$$

$$a(0) = 1 ==> \alpha = 1$$

$$\delta_t = \frac{a'(t)}{a(t)} = \frac{2\beta t}{1 + \beta t^2} ==> \delta_5 = \frac{10\beta}{1 + 25\beta} = 0.08$$

$$10\beta = 0.08 + 2\beta ==> 8\beta = 0.08 ==> \beta = \frac{0.08}{8} = 0.01$$

$$Xa(10) = 25,000 ==> X = \frac{25,000}{a(10)} = \frac{25,000}{1 + 0.01(10)^2} = \frac{25,000}{2} = 12,500$$

4. Wagner Corporation invests 1,000,000 today to build a factory. Wagner expects to have the following cash flows:

Time	Cash Flow
1	Negative 400,000
2	Positive 300,000
3	Positive 500,000
4	X

With this set of cash flows, Wagner expects to have an Internal Rate of Return of 8%.

Calculate the Net Present Value at 10%.

Solution:

$$-1,000,000-400,000(1.08)^{-1}+300,000(1.08)^{-2}+500,000(1.08)^{-3}+X(1.08)^{-4}=0$$

$$X=974,453.76$$

$$NPV=-1,000,000-400,000(1.10)^{-1}+300,000(1.10)^{-2}+500,000(1.10)^{-3}+974,453.76(1.10)^{-4}$$

$$=-74,480.05$$

5. Seamus loans Gage 8000. Gage will repay the loan with three annual payments of 3000. Seamus reinvests each payment at an annual rate of $r\%$.

   After taking reinvestment into account, Seamus has an annual return of 8%.

   Determine r.

   Solution:

$$8000(1.08)^3 = 3000(1+i)^2 + 3000(1+i) + 3000$$

Let $x = 1+i$

$$10,077.696 = 3000x^2 + 3000x + 3000$$

$$3000x^2 + 3000x - 7077.696 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3000 \pm \sqrt{(3000)^2 - 4(3000)(-7077.696)}}{(2)(3000)} = 1.115311735$$

$$i = 0.11531$$

6. Mattie is the beneficiary of a perpetuity due with quarterly payments of 1000. The present value of Mattie's perpetuity is 40,000.

   Mitch borrows 500,000 which will be repaid with quarterly payments of 16,000 plus a drop payment. The interest rate on the loan is the same as the interest rate on Mattie's perpetuity.

   Calculate the drop payment on Mitch's loan.

   Solution:

   Mattie

   Note that it is a perpetuity due with quarterly payments

$$1000\left(\frac{1}{\frac{i^{(4)}}{4}}\right)\left(1+\frac{i^{(4)}}{4}\right)=40,000 ==> \left(\frac{1000}{\frac{i^{(4)}}{4}}\right)+1000=40,000 ==> \frac{i^{(4)}}{4}=\frac{1}{39}=0.025641026$$

   Mitch

   We note that Mitch's payments are also quarterly so we just use $\frac{i^{(4)}}{4}$.

$\boxed{I/Y} \leftarrow 2.5641026$ <== Note that this must go in as a percent and not as a decimal.

$\boxed{PV} \leftarrow 500,000$

$\boxed{PMT} \leftarrow -16,000$

$\boxed{CPT}\boxed{N} \Rightarrow 63.823$ Round down to 63

$\boxed{2nd}\boxed{Amort}\boxed{P1} \leftarrow 1 \boxed{P2} \leftarrow 63 \boxed{\downarrow} \boxed{Bal} \Rightarrow 12,873.81$

$Drop = (12,873.81)(1.025641026) = 13,203.91$

7. Sam has a choice of the following two investments:

    a. A United States Treasury Bill which matures in 275 days for 100,000. This Treasury Bill has a price of 95,000.

    b. A Canadian Treasury Bill that has the same quoted rate as the United States Treasury Bill. The Canadian Treasury Bill matures for 110,000 at the end of 170 days.

    Determine the price of the Canadian Treasury Bill.

    Solution:

    US

$$QR = \frac{360}{275} \cdot \frac{100,000 - 95,000}{100,000} = 0.065454545$$

Canadian

$$QR = \frac{365}{170} \cdot \frac{110,000 - P}{P} = 0.065454545$$

$$\frac{110,000 - P}{P} = 0.030485678$$

$$110,000 - P = 0.030485678P$$

$$P = \frac{110,000}{1.030485678} = 106,745.78$$

8. Yue has a loan of 50,000 which is being repaid with non-level payments over 15 years. The first payment is 4000 at the end of one year. The second payment is 8000 at the end of two years. The third payment of 5000 at the end of three years. The fourth payment of 9000 at the end of four years.

   The annual effective interest rate on the loan is 5%.

   Determine the outstanding loan balance right after the payment of 5000 at the end of three years.

   Solutions:

   $$OLB_3 = \text{Accumulated Value of Past Cash Flows}$$

   $$= (50,000)(1.05)^3 - 4000(1.05)^2 - 8000(1.05)^1 - 5000 = 40,071.25$$

9. Reagan, Danial and Claire enter into a financial arrangement. Under this arrangement, Reagan will pay Danial 10,000 today. Danial will pay Regan P at the end of 3 years. Danial will also pay Claire 6000 at the end of 4 years. Claire will pay Reagan P at the end of 8 years.

   Using the bottom line approach, Reagan's annual effective interest rate on her investment is 6%.

   Determine P.

   Solution:

   Reagan's cashflows are $-10,000$ at time 0, P at time 3 and P at time 8.

   Her equation of value is

   $$10{,}000(1.06)^8 = P(1.06)^5 + P$$

   $$P = \frac{10{,}000(1.06)^8}{(1.06)^5 + 1} = 6816.49$$

10. Emma had 30,000 in her investment account on January 1, 2017.

On August 1, 2017, Emma's account had a value of 32,000 and she withdrew 15,000 to pay her tuition.

On January 1, 2018, her account had a value of 18,000. On this date, Emma took all her Christmas gifts and deposited 12,000 into her account.

On December 31, 2018, Emma's account was worth 33,000.

Estimate Emma's annual dollar weighted return using simple interest.

Solution:

$$A + C + I = B \Longrightarrow 30,000 - 15,000 + 12,000 + I = 33,000$$

$$I = 6000$$

$$j = \frac{I}{A + \sum C_t(1-t)} = \frac{6000}{30,000 - 15,000(1 - 7/24) + 12,000(1 - 12/24)} = 0.236453201$$

But we need the annual estimated dollar weighted rate which is i

$1 + i = (1 + j)^{1/T}$ where T is the time being analyzed in years.

$$1 + i = (1.236453201)^{1//2} = 1.111959 \Longrightarrow i = 0.11196$$

11. Nick Bank makes 3 ½ year loans to college seniors. Nick wants to earn an annual rate of 3.2% compounded continuously in order defer consumption. Additionally, Nick expects inflation to be an annual rate of 2.9% compounded continuously during the loan. Finally, since the inflation rate over the time of the loan is unknown, Nick wants to earn an additional annual rate of 0.4% compounded continuously as compensation of the uncertainty of the inflation rate.

    College seniors as a group have a default rate of 8%. For those that default, the recovery rate is 40%. Nick includes default costs in his total interest rate charged.

    Sneha is a college senior and borrows 28,000 from Nick Bank. At the end of 3 ½ years, Sneha repays the loan.

    Determine the amount that Sneha will pay at the end of 3 ½ years to repay the loan.

    Solution:

$$R = 0.032 + 0.029 + 0.004 + s = 0.065 + s$$

$$28,000e^{0.065(3.5)} = 28,000(0.92)e^{(0.065+s)(3.5)} + 28,000(0.08)(0.40)e^{(0.065+s)(3.5)}$$

$$e^{0.065(3.5)} = (0.92)e^{(0.065+s)(3.5)} + (0.08)(0.40)e^{(0.065+s)(3.5)} = 0.952e^{(0.065+s)(3.5)}$$

$$\frac{e^{0.065(3.5)}}{0.952} = e^{(0.065+s)(3.5)} ==> \ln\left[\frac{e^{0.065(3.5)}}{0.952}\right] = \ln\left[e^{(0.065+s)(3.5)}\right]$$

$$0.276690244 = (0.065 + s)(3.5) = 0.2275 + 3.5s$$

$$s = \frac{0.276690244 - 0.2275}{3.5} = 0.014054355$$

$$R = 0.065 + 0.014054355 = 0.079054355$$

$$Amount = 28,000e^{(0.079054355)(3.5)} = 36,925.22$$

12. Carolyn borrows 30,000 to buy a car. The loan has deferred payments. Under the loan, Carolyn will make 60 payments. However, the first payment that Carolyn will make will be at the end of the fourth month.

The loan has an interest rate of 9% compounded monthly.

Determine the amount of Carolyn's loan payment.

Solution:

Payments are monthly so we need $\frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075$

$$30,000 = v^3 Q a_{\overline{60|}}$$

It is v^3 because $Qa_{\overline{60|}}$ is the value one period before the first payment so it is the value at time 3.

$$30,000 = (1.0075)^{-3}(Q)\left(\frac{1-(1.0075)^{-60}}{0.0075}\right)$$

$$Q = \frac{30,000}{(1.0075)^{-3}\left(\frac{1-(1.0075)^{-60}}{0.0075}\right)} = 636.87$$