MATH 373
Test 3
Spring 2019
May 4, 2019

1. ROPS Corporation issues a 25 year callable bond with a par and maturity value of 100,000. The bond has semi-annual coupons at a rate of 8% convertible semi-annually.

   The bond is callable at the end of 10 years. The call value at the end of 10 years is 119,500.

   The bond is callable at the end of 15 years. The call value at the end of 15 years is 114,500.

   Calculate the price of this bond to ensure a yield of 6% convertible semi-annually.

Solution:

N	I/Y	PMT	FV	CPT PV
(10)(2) = 20	6%/2 = 3	(100,000)(0.08/2) = 4000	119,500	125,674.15
(15)(2) = 30	3	4000	114,500	125,574.25
(25)(2) = 50	3	4000	100,000	125,729.76

We select the lowest price so Price = 125,574.25

2. Danielle buys a 20 year bond issued by Ryan Corporation. The bond has a par value of 10,000 and a maturity value of 12,000. The bond pays semi-annual coupons of 450. Danielle bought the bond to yield $X\%$ convertible semi-annually.

   The book value after the 15th coupon is 13,988.06. The amortization of premium in the 16th coupon is 45.72.

   Determine X .

Solution:

$Coupon = 450$ $\quad BV_{15} = 13{,}988.06$ $\quad P_{16} = 45.72$ $\quad r = \frac{X}{2}\%$

$$I_{16} = Coupon - P_{16} = 450 - 45.72 = 404.28$$

$$I_{16} = (BV_{15})r = (BV_{15})\left(\frac{X}{2}\right) = 13{,}988.06\left(\frac{X}{2}\right) = 6994.03X$$

$$X = \frac{404.28}{6994.03} = 0.057803584$$

3. The stock of Irwin Investment Company sells for 256.25. The stock pays quarterly dividends with the next dividend due in 3 months. The dividend payable in three months is expected to be 4.00. Dividends thereafter are expected to increase with the second dividend at the end of 6 months being 4.25 and the dividend at the end of 9 months being 4.50. The dividends will continue to increase with each dividend being 0.25 greater than the previous dividend.

   The stock of Irwin is expected to yield an annual effective yield rate of i based on the dividend discount method.

   Determine i.

Solution:

$$Price = PV = 256.25 = \frac{4}{\frac{i^{(4)}}{4}} + \frac{0.25}{\left(\frac{i^{(4)}}{4}\right)^2} \Rightarrow 256.25\left(\frac{i^{(4)}}{4}\right)^2 - 4\left(\frac{i^{(4)}}{4}\right) - 0.25 = 0$$

$$\frac{i^{(4)}}{4} = \frac{-b \pm \sqrt{(b)^2 - (4)(a)(c)}}{(2)(a)} = \frac{4 \pm \sqrt{(-4)^2 - 4(256.25)(-0.25)}}{2(256.25)} = 0.04$$

$$i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.04)^4 - 1 = 0.16985856$$

4. Guilia decides to buy two stocks. Both stocks are bought to yield 20% compounded quarterly.

   The first stock is the preferred stock of Gao Corporation. This stock pays a dividend of 10 each quarter with the next dividend being payable in two months.

   The second stock is the common stock of Thompson Corporation. The next dividend for Thompson Corporation is paid at the end of three months and will be 8.50. Thompson is a rapidly growing company and dividends are expected to increase in the future with each dividend being 103% of the prior dividend. In other words, the first dividend will be 8.50, the second dividend will be 8.50(1.03), the third dividend will be $8.50(1.03)^2$, etc.

   Using the dividend discount method, calculate the total amount that Guilia will pay to purchase these two stocks.

Solution:

Since payments are made quarterly, we need $\frac{i^{(4)}}{4}$ and we have $i^{(4)}$.

$$\frac{i^{(4)}}{4} = \frac{0.20}{4} = 0.05$$

$$Stock\ 1: Price = PV = \left(\frac{10}{0.05}\right)(1.05)^{1/3} = 203.28$$

We multiply by $(1.05)^{1/3}$ because the next dividend is due in 2 months.

$$Stock\ 2: Price = PV\ = 8.50(1.05)^{-1} + 8.05(1.03)(1.05)^{-2} + 8.50(1.03)^2(1.05)^{-3} + \cdots$$

$$= \left(\frac{8.50(1.05)^{-1} - 0}{1 - (1.03)(1.05)^{-1}}\right) = 425.00$$

$$Total\ Price = 203.28 + 425.00 = 628.28$$

5. Luke can buy the following two bonds:

   a. Bond A is a one year bond with a price of 1020. The bond has annual coupons of 90 and a maturity value of 1000.

   b. Bond B is a two year bond with a maturity value of 10,000. The bond has annual coupons of 700. The price of the bond is 9900.

   Based on these two bonds, determine $f_{[1,2]}$.

   (Hint: Use bootstrapping to find the spot rates and then find the forward rate.)

Solution:

First, we need to find the spot rates using bootstrapping. Then, using the spot rates, we will find the forward rate.

$$Price\ of\ A = 1020 = \frac{1090}{1+r_1} \Rightarrow r_1 = \frac{1090}{1020} - 1 = 0.068627451$$

$$Price\ of\ B = 9900 = \frac{700}{1+r_1} + \frac{10{,}700}{(1+r_2)^2}$$

$$\frac{10{,}700}{(1+r_2)^2} = 9900 - \frac{700}{1.068627451} = 9244.95 \Rightarrow r_2 = \left(\frac{10{,}700}{9244.95}\right)^{0.5} - 1 = 0.075819739$$

$$f_{[1,2]} = \frac{(1+r_2)^2}{1+r_1} - 1 = \frac{(1.075819739)^2}{1.068627451} - 1 = 0.083060434$$

6. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	5.00%	3.0	6.60%
1.0	5.40%	3.5	6.85%
1.5	5.75%	4.0	7.05%
2.0	6.05%	4.5	7.20%
2.5	6.35%	5.0	7.30%

Grigor is receiving an annuity immediate with two annual payments of 10,000.

Using the spot interest rates, you determine the present value of the annuity. You then determine the equivalent annual yield rate on the annuity.

What was the annual yield rate that you determined?

Solution:

$$PV = 10{,}000(1+r_1)^{-1} + 10{,}000(1+r_2)^{-2} = 10{,}000(1.054)^{-1} + 10{,}000(1.0605)^{-2}$$

$$PV = 18{,}379.24018$$

We want to find i , the annual yield rate.

$$18{,}379.24018 = 10{,}000\, a_{\overline{2|}}$$

Use your calculator to find i :

$$\boxed{N} \leftarrow 2\ ;\ \boxed{PV} \leftarrow 18{,}379.24018\ ;\ \boxed{PMT} \leftarrow -10{,}000$$

$$\boxed{CPT}\ \boxed{I/Y} \leftarrow 5.824018766\%$$

7. Kimberly has agreed to pay Aashima a payment of 25,000 at the end of one year and an additional payment of 42,000 at the end of two years.

   In order to protect herself from interest rate changes, Kimberly wants to absolutely match the cash flows using the following two bonds:

   a. Bond 1 is a one year bond with annual coupons of 50 and a maturity value of 1200. The price of this bond is 1150.

   b. Bond 2 is a two year bond with annual coupons of 100 and a maturity value of 2000. The annual effective yield rate on this bond is 5%.

   Determine the number of each of the bonds that Kimberly will need to purchase. Assume that Kimberly can buy partial bonds.

Solution:

$$Time\ 1 \Rightarrow (Bond1)(1{,}250) + (Bond2)(100) = 25{,}000$$

$$Time\ 2 \Rightarrow (Bond1)(0) + (Bond2)(2{,}100) = 42{,}000$$

$$Using\ Time\ 2 \Rightarrow Bond2 = \frac{42{,}000}{2{,}100} = 20$$

$$Using\ Time\ 1 \Rightarrow Bond1 = \frac{25{,}000 - (20)(100)}{1{,}250} = 18.4$$

8. Matt owns a 20 year bond with annual coupons of 1000 and a maturity value of 20,000.

Calculate the Modified Duration of this bond at an annual effective interest rate of 8%.

Solution:

$$MacDur = \frac{\Sigma C_t(t)v^t}{\Sigma C_t v^t}$$

$$= \frac{1{,}000(1)v^1 + 1{,}000(2)v^2 + 1{,}000(3)v^3 + \cdots + 1{,}000(20)v^{20} + 20{,}000(20)v^{20}}{1{,}000\, a_{\overline{20}|0.08} + 20{,}000(1.08)^{-20}}$$

$$= \frac{1{,}000v^1 + 2{,}000v^2 + 3{,}000v^3 + \cdots + 20{,}000v^{20} + 400{,}000v^{20}}{1{,}000\, a_{\overline{20}|0.08} + 20{,}000(1.08)^{-20}}$$

$$= \frac{1{,}000(1.08)^{-1} + 2{,}000(1.08)^{-2} + 3{,}000(1.08)^{-3} + \cdots + 20{,}000(1.08)^{-20} + 400{,}000(1.08)^{-20}}{1{,}000\, a_{\overline{20}|0.08} + 20{,}000(1.08)^{-20}}$$

$$= \frac{1{,}000\left(\frac{1-(1.08)^{-20}}{0.08}\right) + \left(\frac{1{,}000}{0.08}\right)\left(\frac{1-(1.08)^{-20}}{0.08} - 20(1.08)^{-20}\right) + 400{,}000(1.08)^{-20}}{1{,}000\left(\frac{1-(1.08)^{-20}}{0.08}\right) + 20{,}000(1.08)^{-20}}$$

$$= 11.67523699$$

$$ModDur = v(MacDur) = (1.08)^{-1}(11.6752) = 10.8104$$

9. Christopher has agreed to pay Anthony an annuity due with four payments of 1000. The payments are at the beginning of each 6 months period for two years.

   Calculate the Macaulay Convexity of this annuity at an annual effective interest rate of 6%.

Solution:

Since payments are made semiannually, we need $\frac{i^{(2)}}{2}$ and we have i.

$$\frac{i^{(2)}}{2} = (1+i)^{\frac{1}{2}} - 1 = (1.06)^{\frac{1}{2}} - 1 = 0.029563$$

$$MacCon = \frac{\Sigma C_t (t^2) v^t}{\Sigma C_t v^t}$$

$$= \frac{1{,}000(0)v^0 + 1{,}000(0.5)^2 v^1 + 1{,}000(1)^2 v^2 + 1{,}000(1.5)^2 v^3}{1{,}000\,\ddot{a}_{\overline{4|}0.029563}}$$

$$= \frac{1{,}000(0.5)^2(1.06)^{-0.5} + 1{,}000(1)^2(1.06)^{-1} + 1{,}000(1.5)^2(1.06)^{-1.5}}{1{,}000\,\ddot{a}_{\overline{4|}0.029563}}$$

$$= \frac{1{,}000(0.5)^2(1.06)^{-0.5} + 1{,}000(1)^2(1.06)^{-1} + 1{,}000(1.5)^2(1.06)^{-1.5}}{1{,}000\left(\frac{1-(1.029563)^{-4}}{0.029563}\right)(1.029563)}$$

$$= 0.84779908$$

10. You have the following portfolio of bonds:

    a. A zero coupon bond which matures for 60,000 at the end of 9 years.

    b. A 12 year bond with a Modified Convexity of 80 and a price of 40,000.

Calculate the Modified Convexity for this portfolio of bonds at an interest rate of 5%.

Solution:

$\underline{Bond\ A:}$

$Since\ this\ is\ a\ zero\ coupon\ bond.$

$MacDur = 9\,, \qquad MacCon = 9^2 = 81$

$$ModCon = (MacDur + MadCon)v^2 = \frac{(9+81)}{(1.05)^2} = 81.63265306$$

$$Price = \frac{60{,}000}{(1.05)^9} = 38{,}676.53$$

$\underline{Bond\ B:}$

$ModCon = 80, \qquad Price = 40{,}000$

$$C_{ModCon}^{Port} = \frac{\Sigma P^t Ct}{\Sigma P^t} = \frac{(38{,}676.53)(81.63265306) + (40{,}000)(80)}{38{,}676.53 + 40{,}000}$$

$$= 80.8026$$

11. Brad has a portfolio of bonds which have a current price of 1,000,000. The Macaulay duration of this portfolio is 8 and the Macaulay convexity of the portfolio is 52. These values are all based on an interest rate of 7.5%.

P^{Mac} is estimated price of this portfolio using the first order Macaulay approximation with the interest rate is 6.5%.

P^{Mod} is estimated price of this portfolio using the second order Modified approximation with the interest rate is 6.5%.

Calculate $P^{Mac} - P^{Mod}$.

Solution:

First Order Macaulay Approximation:

$$P^{Mac} = P(i) \approx P(i_0)\left[\frac{1+i_0}{1+i}\right]^{MacDur}$$

$$P^{Mac} \approx (1{,}000{,}000)\left[\frac{1.075}{1.065}\right]^{8} = 1{,}077{,}632.915$$

Second Order Modified Approximation:

$$P^{Mod} = P(i) \approx P(i_0)[1 - (i - i_0)(ModDur) + \frac{(i - i_0)^2}{2}(ModCon)]$$

$$ModDur = v(MacDur) = 1.075^{-1}(8) = 7.44$$

$$ModCon = v^2(MacDur + MacCon) = 1.075^{-2}(8 + 52) = 51.92$$

$$P^{Mod} \approx (1{,}000{,}000)[1 - (0.065 - 0.075)(7.44) + \frac{(0.065 - 0.075)^2}{2}(51.92)]$$

$$= 1{,}077{,}014.602$$

$$P^{Mac} - P^{Mod} = 1{,}077{,}632.915 - 1{,}077{,}014.602 = 618.323$$

12. The Thea Insurance Company has agreed to pay 500,000 to Karinna at the end of six years. The annual effective interest rate is 4%. In order to protect against interest rate changes, Thea has decided to fully immunize the payment using the following to zero coupon bonds:

    a. Bond A matures for 10,000 at the end of 3 years.

    b. Bond B matures for 25,000 at the end of 7 years.

    Determine the number of Bond B that Thea should purchase. Assume that Thea can buy partial bonds.

Solution:

$$AmountBondB = \frac{D^L - D^A}{D^B - D^A}(PV\ of\ Liability) = \left(\frac{6-3}{7-3}\right)(500{,}000)(1.04)^{-6}$$

$$= 296{,}367.9471$$

$$NumberBondB = \frac{AmountBondB}{PriceBondB} = \frac{296{,}367.9471}{(25{,}000)(1.04)^{-7}} = 15.6$$

You are given the following spot interest rates and information for questions 13 and 14:

t	r_t	t	r_t
0.5	5.00%	3.0	6.60%
1.0	5.40%	3.5	6.85%
1.5	5.75%	4.0	7.05%
2.0	6.05%	4.5	7.20%
2.5	6.35%	5.0	7.30%

Nick Bank makes a loan to Watkins Corporation. The loan is a three year loan for 500,000. The loan has a variable interest rate which is equal to the one year spot rate at the beginning of each year of the loan. Watkins will pay the interest on the loan at the end of each year for three years. Additionally, Watkins will pay the principal of the loan which is 500,000 at the end of the third year.

Watkins is worried about interest rates changing and would like to have a fixed interest rate instead of a variable interest rate. Therefore, Janson Investment Bank and Watkins Corporation enter into an interest rate swap. Under the swap, Janson will pay a variable interest rate to Watkins and Watkins will pay the fixed interest rate to Janson. The terms of the swap mirror the terms of the loan.

13. Answer the following four questions which will count as one question on the exam:

   a. Who is the payer in this scenario? **Watkins Corporation**

   b. What is the Swap Tenor? **Three Years**

   c. Which entity is not a counterparty to this Swap? **Nick Bank**

   d. State the Notional Amount for this Swap. **500,000**

14. Determine the Swap Rate to five decimal places for this Swap.

Solution:

$$R = \frac{1 - P_3}{P_1 + P_2 + P_3} = \frac{1 - (1.0660)^{-3}}{(1.0540)^{-1} + (1.0605)^{-2} + (1.0660)^{-3}} = 0.065508569 \approx 0.06551$$

15. You are given the following spot interest rates:

t	r_t	t	r_t
0.5	5.00%	3.0	6.60%
1.0	5.40%	3.5	6.85%
1.5	5.75%	4.0	7.05%
2.0	6.05%	4.5	7.20%
2.5	6.35%	5.0	7.30%

Mueller Corporation and Kellyn Bank enter into an interest rate swap. The swap is a five year swap with annual periods. However, the swap is a deferred swap and there is no swap of interest rates during the first three years. The swap has a notational amount of 1,300,000 during the fourth year and 900,000 during the fifth year.

Determine the swap interest rate to five decimal places under this swap.

Solution:

$$R = \frac{\Sigma Q \cdot f \cdot P}{\Sigma Q \cdot P}$$

$$(1.066)^3\left(1 + f_{[3,4]}\right) = (1.0705)^4 \Rightarrow f_{[3,4]} = \frac{(1.0705)^4}{(1.066)^3} - 1 = 0.084114299$$

$$(1.0705)^4\left(1 + f_{[4,5]}\right) = (1.0730)^5 \Rightarrow f_{[4,5]} = \frac{(1.0730)^5}{(1.0705)^4} - 1 = 0.08305852$$

$$R = \frac{(1{,}300{,}000)(0.084114299)(1.0705)^{-4} + (900{,}000)(0.08305852)(1.0730)^{-5}}{(1{,}300{,}000)(1.0705)^{-4} + (900{,}000)(1.0730)^{-5}}$$

$$= 0.083702596 \approx 0.08370$$

16. Zach and Alex enter into a five year interest rate swap with annual swap period. Under the swap, Zach agrees to pay the fixed interest rate of 6.42% to Alex. In return, Alex agrees to pay the variable interest rate to Zach at the end of each year. The amount of the swap is 8,000,000.

    At the beginning of the second year of the swap, the variable interest rate is 5.79%.

    Determine the net swap payment at the end of the second year and state who makes that payment.

Solution:

Zach will pay fixed rate $\Rightarrow (8{,}000{,}000)(0.0642) = 513{,}600$

Alex will pay variable rate $\Rightarrow (8{,}000{,}000)(0.0579) = 463{,}200$

Net Swap Payment: $513{,}600 - 463{,}200 = 50{,}400$

Zach will make a payment of $50{,}400$ to Alex.