1. For the random variable X, you are given the probability density function:

$$f_X(x) = \begin{cases} cx^2, & 0 \le x \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate the $E[X]$.

Solution:

First find c.

$$\int_0^5 f_X(x) \cdot dx = 1 \Longrightarrow \int_0^5 cx^2 \cdot dx = 1 \Longrightarrow \left[\frac{cx^3}{3}\right]_0^5 = \frac{125c}{3} = 1 \Longrightarrow c = \frac{3}{125}$$

$$E[X] = \int_0^5 x \cdot f_X(x) \cdot dx = \int_0^5 x \cdot \frac{3}{125} x^2 \cdot dx = \frac{3}{125} \int_0^5 x^3 \cdot dx = \left(\frac{3}{125}\right)\left[\frac{x^4}{4}\right]_0^5$$

$$= \left(\frac{3}{125}\right)\left[\frac{5^4}{4}\right] = \frac{15}{4} = 3.75$$

2. A bowl has 11 balls in it. Nine of the balls are purple and two are red. You draw a ball from the bowl and note the color. You then replace the ball in the bowl and draw again.

Let N be the random variable representing number of the draw upon which you get the first purple ball.

Calculate the $E[N]$.

Solution:

$$E[N] = \sum_{n=1}^{\infty} n \cdot p(n) = (1)\left(\frac{9}{11}\right) + (2)\left(\frac{2}{11}\right)\left(\frac{9}{11}\right) + (2)\left(\frac{2}{11}\right)^2\left(\frac{9}{11}\right) + \dots$$

$$= \frac{c}{(1-r)^2} = \frac{\left(\frac{9}{11}\right)}{\left(1-\left(\frac{2}{11}\right)\right)^2} = \frac{11}{9}$$

3. I would like to receive credit for this question because I wrote my name on the cover.

(**True**) or **False**