STAT 416
Quiz 3
Fall 2021

October 21, 2021

1. (10 points) You are given that the random variable X has the following pdf:

$$f_X(x) = \begin{cases} \dfrac{3x^2}{1000}, & 0 \le x \le 10 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate $Var[X]$.

Solution:

$$Var[X] = E[X^2] - \left(E[X]\right)^2$$

$$E[X] = \int_0^{10} x \cdot f_X(x) \cdot dx = \int_0^{10} x \cdot \frac{3x^2}{1000} \cdot dx = \left[\frac{3x^4}{4000}\right]_0^{10} = 7.5$$

$$E[X^2] = \int_0^{10} x^2 \cdot f_X(x) \cdot dx = \int_0^{10} x^2 \cdot \frac{3x^2}{1000} \cdot dx = \left[\frac{3x^5}{5000}\right]_0^{10} = 60$$

$$Var[X] = 60 - (7.5)^2 = 3.75 = \frac{15}{4}$$

2. A bowl has four tiles numbered 1, 2, 3, and 4. You randomly draw a tile from the bowl and note the number. You **REPLACE** the tile and draw again.

   Let the random variable N be the sum of the two tiles.

   a. (10 points) Calculate $E[N]$.

   Solution:

		Draw of First Tile			
		1	**2**	**3**	**4**
Draw of Second Tile	**1**	**2**	**3**	**4**	**5**
	2	**3**	**4**	**5**	**6**
	3	**4**	**5**	**6**	**7**
	4	**5**	**6**	**7**	**8**

   The values in red are the totals of the two tiles.

$$p(2)=\frac{1}{16};\ p(3)=\frac{2}{16};\ p(4)=\frac{3}{16};\ p(5)=\frac{4}{16};\ p(6)=\frac{3}{16};\ p(7)=\frac{2}{16};\ p(8)=\frac{1}{16}$$

$$E[N]=(2)\left(\frac{1}{16}\right)+(3)\left(\frac{2}{16}\right)+(4)\left(\frac{3}{16}\right)+(5)\left(\frac{4}{16}\right)+(6)\left(\frac{3}{16}\right)+(7)\left(\frac{2}{16}\right)+(8)\left(\frac{1}{16}\right)=5$$

   b. (10 points) Calculate $Var[N]$.

   Solution:

$$E[X^2]=(2)^2\left(\frac{1}{16}\right)+(3)^2\left(\frac{2}{16}\right)+(4)^2\left(\frac{3}{16}\right)+(5)^2\left(\frac{4}{16}\right)$$
$$+(6)^2\left(\frac{3}{16}\right)+(7)^2\left(\frac{2}{16}\right)+(8)^2\left(\frac{1}{16}\right)=27.5$$

$$Var[N]=27.5-(5)^2=2.5$$

This problem was solved many different ways. The above is just one of the solutions.