

STAT 416
Quiz 5
Fall 2021
November 23, 2021

1. The random variable $X \sim \text{Gamma}(2, \beta)$ and has mean of 6.

Calculate $E[X^2]$.

Solution:

$$E[X] = \frac{\alpha}{\beta} = \frac{2}{\beta} = 6 \implies \beta = \frac{1}{3}$$

$$\text{Var}[X] = \frac{\alpha}{\beta^2} = E[X^2] - (E[X])^2 \implies \frac{2}{(1/3)^2} = E[X^2] - (6)^2 \implies 18 = E[X^2] - 36$$

$$E[X^2] = 18 + 36 = 54$$

Also

$$E[X^2] = \frac{(\alpha+1)}{\beta^2} = \frac{3}{(1/3)^2} = 54$$

2. The average length of time between airplane arrivals at the airport is modeled as an exponential distribution. The inter-arrival times for airplanes is 3 minutes.

Calculate the probability that the next plane will arrive between five and ten minutes from now.

Solution:

$$\theta = \frac{1}{3}$$

$$\Pr[5 < N < 10] = S(5) - S(10) = e^{-5\theta} - e^{-10\theta} = e^{-5/3} - e^{-10/3} = 0.1532$$

3. The random variable $X \sim \text{Normal}(7, 64)$.

Calculate the $\Pr[-1 < X < 9]$.

Solution:

$$\Pr[-1 < X < 9] = \Pr\left[\frac{-1-7}{\sqrt{64}} < \frac{X-\mu}{\sigma} < \frac{9-7}{\sqrt{64}}\right] = \Pr[-1 < Z < 0.25]$$

$$= \Phi(0.25) - \Phi(-1) = \Phi(0.25) - \{1 - \Phi(1)\} = 0.5987 - \{1 - 0.8413\} = 0.440$$