STAT 416
Quiz 6
Fall, 2021

December 2, 2021

1. You are given that the Random Variable X can have values of 2, 4, 6 and random variable Y can have values of 1, 2, 5, 6. Further, the table below lists the joint probability mass function.

	Y			
X	1	2	5	6
2	1/100	5/100	9/100	13/100
4	2/100	6/100	10/100	14/100
6	3/100	7/100	11/100	19/100

a. Calculate the marginal pdf of X .

Solution:

$$p_X(2) = \frac{1}{100} + \frac{5}{100} + \frac{9}{100} + \frac{13}{100} = \frac{28}{100}$$

$$p_X(4) = \frac{2}{100} + \frac{6}{100} + \frac{10}{100} + \frac{14}{100} = \frac{32}{100}$$

$$p_X(8) = \frac{3}{100} + \frac{7}{100} + \frac{11}{100} + \frac{19}{100} = \frac{40}{100}$$

b. Calculate $F_{X,Y}(4,5)$.

Solution:

$$F_{X,Y}(4,5) = \Pr(X \le 4, Y \le 5)$$

$$= p_{X,Y}(2,1) + p_{X,Y}(2,2) + p_{X,Y}(2,5) + p_{X,Y}(4,1) + p_{X,Y}(4,2) + p_{X,Y}(4,5)$$

$$= \frac{1}{100} + \frac{5}{100} + \frac{9}{100} + \frac{2}{100} + \frac{6}{100} + \frac{10}{100} = \frac{33}{100}$$

2. The number of students riding an elevator in the Math Building in an hour is a Poisson distribution with a mean of 25.

   Let N be the random variable representing the number of students who ride the elevator in a 24 hour period.

   Use the Central Limit Theorem to estimate $\Pr[550 < N \le 650]$. Calculate this probability using the continuity correction.

   Solution:

$$\lambda = (25)(24) = 600$$

$$\Pr[550 < N \le 650] \approx \Pr[550.5 < N \le 650.5]$$

$$= \Pr\left[\frac{550.5 - 600}{\sqrt{600}} < \frac{N - 600}{\sqrt{600}} \le \frac{650.5 - 600}{\sqrt{600}}\right]$$

$$\Pr\left[-2.02 < Z \le 2.06\right] = \Phi(2.06) - [1 - \Phi(2.02)]$$

$$= \Phi(2.06) + \Phi(2.02) - 1 = 0.9803 + 0.9783 - 1$$

$$= 0.9586$$

3. Let $X_1, X_2, ..., X_n$ be an random sample from a distribution with mean $\mu_X = 60$ and variance of 400.

Using the Central Limit Theorem, determine the number of samples necessary to be 90% certain that $\bar{X}_n$ is within 2 of the true value of μ_X.

Solution:

$$\Pr\left[\frac{\bar{X}_n - 2 - \mu_X}{\sigma_x / \sqrt{n}} < Z < \frac{\bar{X}_n + 2 - \mu_X}{\sigma_x / \sqrt{n}}\right] = 0.90$$

$$\Pr\left[\frac{-2}{20/\sqrt{n}} < Z < \frac{2}{20/\sqrt{n}}\right] = 0.90$$

$$\Phi\left[\frac{2\sqrt{n}}{20}\right] - \left[1 - \Phi\left[\frac{2\sqrt{n}}{20}\right]\right] = 0.90$$

$$2\Phi\left[\frac{2\sqrt{n}}{20}\right] - 1 = 0.9 ==> 2\Phi\left[\frac{2\sqrt{n}}{20}\right] = 1.9 ==> \Phi\left[\frac{2\sqrt{n}}{20}\right] = 0.95$$

$$\frac{2\sqrt{n}}{20} = 1.645 ==> n = \left(\frac{1.645(20)}{2}\right)^2 = 270.6025 ==> n = 271$$