1. You complete an experiment by rolling a six sided fair die.

   The following events are defined:

      i. A = {1, 2, 3}
      ii. B = {4, 5, 6}
      iii. C = {1}
      iv. D = {1, 3, 5}
      v. E = {2, 4, 6}

   a. Determine $A \bigcup D$
   b. Determine $A \bigcap D$
   c. Determine $A \bigcap B$
   d. Determine A^C
   e. Determine $B^C \bigcap C$
   f. Determine $(B \bigcup C)^C$
   g. State whether the following statements are true of false. If false, state why it is false.
      1. Events A and B are disjoint events.
      2. Events A, B, and C are mutually disjoint events.
      3. $D \bigcap E$ is an empty set, but not a null set
      4. $C \subset A$

2. You are given:

      i. $P[A] = 0.5$
      ii. $P[B] = 0.4$
      iii. $P[A \bigcap B] = 0.3$

   a. Calculate $P[A \bigcup B]$
   b. Calculate $P[A^C \bigcup B^C]$
   c. Calculate $P[A^C \bigcap B^C]$
   d. Calculate $P[A^C \bigcap B]$
   e. Calculate $P[A^C \bigcup B]$

3. Let E and F be events such that $P[E]=1/2,\ P[F]=1/2,$ and $P[E^C \cap F^C]=1/3.$

   Calculate $P[E \cup F^C]$.

4. You are given $P[A \cup B]=0.7$ and $P[A \cup B^C]=0.9$. Calculate $P[A]$.

5. A marketing survey indicates that 60% of the population own an automobile, 30% own a house, and 20% own both. Calculate the probability that a person chosen at random owns an automobile or a house but not both.

6. A visit to a doctor's office results in a lab work 40% of the time. A visit to a doctor's office results in a referral to a specialist 30% of the time. A visit to a doctor's office results in no lab work and no specialist referral 35% of the time.

   What is the probability that a visit to the doctor's office will result in both lab work and a referral to a specialist?

7. An employer offers employees the following coverages:

   a. Major Medical Coverage

   b. Vision Coverage

   c. Dental Coverage.

   Employees who enroll are required to enroll in at least two coverages. You may elect to have zero coverages. You are given:

   a. The probability of enrolling in Major Medical is 80%.

   b. The probability of enrolling in Vision is 40%

   c. The probability of enrolling in Dental is 70%

   d. The probability of enrolling in all three is 20%

   Calculate the probability of enrolling in zero coverages.

8. In a survey of 120 high school students, the following data was obtained:

   a. 60 students participated in Cross Country
   b. 56 students participated in Track
   c. 42 students participated in Golf
   d. 34 students participated in both Cross Country and Track
   e. 20 students participated in both Track and Golf
   f. 16 students participated in Cross Country and Golf
   g. 6 students participated in all three sports

   Calculate:

   i. The number of students who did not participate in any of these sports.
   ii. The number of students who did Track only.
   iii. The number of students who did Cross Country and Track, but not Golf.

9. You roll two fair dice. Calculate the probability of the following events:

   a. Total of the two dice is 8.
   b. You roll a 4 and a 2.
   c. The total of the two dice is less than 6.
   d. The number on each die is the same.

10. A license plate contains a letter followed by five numeric digits that can be 0 to 9.

    a. What is the size of the sample space if digits can be repeated.
    b. What is the size of the sample space if digits cannot be repeated.

11. A bowl has five balls. They are red, green, blue, purple, and yellow. You draw all five balls out of the bowl.

    How many possible orders are there?

12. A bowl has five balls. They are red, green, blue, purple, and yellow. You draw all three balls out of the bowl without replacement.

    How many possible orders are there?

13. A bowl has five tiles which each tile being numbered. The tiles are numbered 1 to 5. You draw three tile balls out of the bowl without replacement.

    What is the probability of getting a number greater than 350?

14. A bowl has 10 balls. Four balls are purple, three balls are red, two balls are blue and one ball is green. You draw three balls without replacement.

    a. Determine the number of unique arrangements.

    b. Determine the probability that you will draw 2 purple and one red ball.

15. A bowl contains 4 blue, 5 white, 6 red and 7 green balls. You chose a sample of 5 balls without replacement. How many different ways can you select a sample that has every color represented?

16. A softball team has 10 players which consists of five males and five females. All players bat. In determining a batting order, a female must bat first. Furthermore, successive batters must be of the opposite gender.

    Determine the number of possible batting orders.

17. A committee of five people is randomly chosen from a list of that contains 7 men and 3 females.

    Calculate the probability that the committee will have 3 men and 2 women?

18. What is the probability that a hand of five cards chosen at random and without replacement from a standard 52 card deck will have a king of spades, exactly one other king, and exactly two queens?

19. A bowl has 6 balls. Three balls are red, two balls are blue and one ball is green. You randomly draw three balls without replacement.

    Determine the probability that at least one color is **not** drawn.

20. A bowl has 10 balls. Five balls are purple, three balls are red and two balls are blue. You randomly draw three balls **with** replacement.

    What is the probability that the three balls are include one of each color?

21. If four fair die are rolled, what is the probability of obtaining two identical even numbers and two identical odd numbers.

22. An employer has 30 employees. There are 5 employees that are older than 60 and 25 employees that are 60 or younger. Two employees are selected at random.

    What is the probability that exactly one of the two selected employees is over 60?

23. In a neighborhood with 10 houses, k houses are not insured.

    A tornado randomly damages 3 of the houses. The probability that none of the damaged houses are insured is 1/120.

    Calculate the probability that at most one of the damaged houses is insured.

24. A professor approaches 12 undergraduate students independently, each of whom is equally likely to want to participate in research projects. Six are interested only in applied statistics research, four are interested only in actuarial science research, and two are interested only in data science research.

    After the talking with the students, six students state that they want to participate in the research.

    Calculate the probability that two participate in applied statistics research, two participate in actuarial research, and 2 participate in data science research.

25. Roll a six sided fair die. Let A be the event that the outcome on the dice is an even number. Let B be the event that the outcome on the dice is 4 or smaller. Let C be the event that the outcome on the dice is 3 or larger.

    a. Are A and B independent?

    b. Are B and C independent?

26. You roll two six sided fair die – one is red and one is green. Event A is that the red die is a 6. Event B is that the sum of the two die exceeds 8.

    a. Demonstrate that these events are not independent.

    b. Calculate $P(B|A)$.

27. A dresser drawer contains five gloves – One pair of blue gloves, one pair of red gloves, and a single white glove. Suppose that Olivia is looking for two matching blue gloves.

    Olivia randomly simultaneously pulls two gloves out of the drawer without looking in the drawer. Let Event B be the event that at least one of the gloves is blue. Let Event A be the event that both gloves are blue.

    Calculate the conditional probability of Event A given that Event B occurs.

28. You roll two pair of fair dice. Let B be the event that the two dice have different values. Given the B occurs, calculate the conditional probability of A, the event that the sum of the two dice is an even number.

29. You roll two pair of fair dice. Let B be the event that the sum of the two dice is 9 or larger. Given the B occurs, calculate the conditional probability of A, the event that the sum of the two dice is exactly 10.

30. At Purdue, 40% of the students live on campus and 60% live off campus. Students who live on campus arrive to class on time 85% of the time. Students who live off campus arrive to class on time 70% of the time.

    A randomly selected student arrived on time. What is the probability that the student lives on campus.

31. There are two coins in a hat. One coin has two heads. The other coin has one head and one tail. Shannon reaches into the hat and pulls out a coin. The side facing up is heads. What is the probability that the side facing down is a head also?

32. Let A, B, C be three events. You are given:

    $$P[A] = 0.3 \qquad P[B] = 0.2$$

    $$P[C \mid A] = 0.6 \qquad P[C \mid B] = 0.8$$

    You are also given that $C \cap A$ and $C \cap B$ are mutually exclusive.

    Calculate $P[C \cap (A \cup B)]$.

33. Let A, B, C be three events. The events are pairwise independent but not mutually independent. The probability of each event is 1/3 and the probability that all three occur is 1/10. Calculate the probability that none of them occur.

34. A die is rolled three times in succession. What is the probability that the outcome on each roll is less than that on the previous roll and the product of the three outcomes is an even number?

35. A box contains 4 red balls and 6 white balls. A sample of size 3 is drawn without replacement from the box. What is the probability of obtaining one red ball and two white balls, given that at least 2 white balls are drawn in the sample.

36. Let A, B, C be three events such that A and B are independent, B and C are mutually exclusive, and $P[A] = \frac{1}{4}$, $P[B] = \frac{1}{6}$, and $P[C] = \frac{1}{2}$.

    Calculate $P[(A \cap B)^C \cup C]$.

37. A researcher examines medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease. Of those 312 men, 102 died from causes related to heart disease.

    Calculate the probability that a man randomly selected from this group dies of caused related to heart disease, given that neither of his parents suffered from heart disease.

38. You are given:

    a. An automobile owner is twice as likely to purchase collision coverage as disability coverage.

    b. The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.

    c. The probability that an automobile owner purchases both collision and disability coverage is 15%

    Calculate the probability that an automobile owner purchases neither collision nor disability coverage.

39. You are studying the prevalence of three health risk factors denoted by $A, B \text{ and } C$ within a population of women. For each of the three factors, the probability is 0.1 that a women in the population has this risk but not the other risk factors. For any two of the three factors, the probability is 0.12 that she has exactly these two factors but not the third factor. The probability that a woman has all three risk factors given that she has $A \text{ and } B$ is 1/3.

    Calculate the probability that a woman has none of the three factors given that she does not have factor A .

40. Aiden rolls two six sided die. Let D be the random variable representing the absolute value of the difference between the value of each die. Let M be the random variable representing the minimum value on the two die.

    a. State the possible values of D.

    b. State the possible value of M .

    c. Caclulate $P(D=2)$.

    d. Calculate $P(M=2)$.

    e. Calculate $F_D(d)$.

41. In a company, 10% of the employees in their first five years earn 100,000 or more while 20% of employees with more than 5 years with the company earn 100,000 or more. Overall, 13% of employees earn 100,000 or more.

    The company has 10,000 employees. Determine the number of employees who are in their first five years with the company.

42. In a doctor's practice, 20% of patients have high blood pressure and 30% of patients have high cholesterol. Of the patients with high blood pressure, 25% have high cholesterol.

    A patient is selected at random.

    Calculate the probability that the patient has high blood pressure, given that the patient has high cholesterol.

43. A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease.

    Calculate the probability that a person has the disease given the test indicates the presence of the disease.

44. Mihika takes a test consisting of multiple choice questions where each question has five answer choices. Mihika may or may not know the answer to the question. If she knows the answer, she will answer the question correctly. Otherwise, she will randomly guess the answer.

    For a given question, the conditional probability that that Mihika knows the answer to the question, given that Mihika answered it correctly, is 0.824. Find the probability that Mihika knows the answer to the question.

45. A continuous random variable X has a density function:

$$f_X(x) = \begin{cases} \dfrac{1}{(1+x)^2}, & x \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

    Calculate $P(2 < X < 5)$.

46. For the random variable X, you are given the probability density function:

$$f_X(x) = \begin{cases} cx^2, & 1 \leq x \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

    Calculate $P(2 \leq X \leq 3)$.

47. The distribution of the size of claims paid under an insurance policy has PDF of:

$$f_X(x) = \begin{cases} cx^\alpha, & 0 < x < 5 \\ 0, & \text{elsewhere} \end{cases} \quad \text{for } \alpha > 0 \text{ and } c > 0$$

    For a randomly selected claim, the probability that the size of the claim is less than 3.75 is 0.4871.

    Calculate the probability that the size of a randomly selected claim is greater than 4.

48. The lifetime of a car has a continuous distribution on the interval (0,40) with a PDF that is proportional to $(10+x)^{-2}$ on the interval.

Calculate the probability that the lifetime of the car is less than 6.

49. The lifetime of a car has a continuous distribution on the interval (0,40) with a PDF that is equal to $12.5(10+x)^{-2}$ on the interval.

Determine $S_X(x)$.

50. A discrete random variable N has the following probability function:

$$f_N(n)=\begin{cases} p, & n=1 \\ 2p, & n=3 \\ 1-3p, & n=5 \end{cases}$$

You are also given that $E[N]=3.8$.

Calculate p .

51. John rolls a die and will be paid 10 minus the number on the die as a reward.

Calculate the expected value of John's reward.

52. Calculate $8+16+32+64+...+8192$.

53. Calculate $8+14+20+26+...+596$.

54. Ethan rolls a six sided fair die until he gets a 6. Let N be the random variable that is the number of the roll on which Ethan gets the first 6.

Calculate $E[N]$.

55. The length of a staff meeting, in hours, is a random variable having the following probability density function:

$$f_X(x)=\begin{cases} 5(x-0.5)^4 & 0.5\le x\le 1.5 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the expected length of the staff meeting.

56. Let X be the continuous random variable with density function

$$f_X(x) = \begin{cases} \theta\text{x} + 1.5\theta^{1.5}x^2 & 0 \le x \le 1/\sqrt{\theta} \\ 0, & \text{otherwise} \end{cases}$$

Where $\theta > 0$.

What is the expected value of X in terms of θ .

57. Let X be a continuous random variable with density function:

$$f_X(x) = \begin{cases} \dfrac{|x|}{10} & -2 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

What is the expected value of X .

58. Let X be a continuous random variable with a density function:

$$f_X(x) = \begin{cases} \dfrac{p-1}{x^p}, & x > 1 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the value of p such that $E[X] = 2.$

59. (Review Question) A box contains 10 balls, of which 3 are red, two are yellow and 5 are blue. Five balls are randomly selected with replacement.

    Calculate the probability that fewer than 2 of the selected balls are red.

60. (Review Question) A plane has 30 seats. The probability that an particular passenger does not show up for a flight is 10% and independent of other passengers. The airline sells 32 tickets for a flight.

    Calculate the probability that more passengers show up for the flight that there are seats.

61. (Review Question) Defective items on an assembly line occur independently with a probability of 0.05. A random sample of 100 items are selected. Calculate the probability that the first item sampled is not defective given that at least 99 of the items are not defective.

62. Let X be a continuous random variable with density function

$$f_X(x) = \begin{cases} \frac{1}{30}x(1+3x), & 1 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

Calculate $E[1/X]$.

63. You are given two random variables X and Y.

X is has a probability mass function of:

$$\begin{aligned} &0.2 \ \text{ if } x=3 \\ &0.3 \ \text{ if } x=8 \\ &0.5 \ \text{ if } x=10 \\ &0 \quad \text{elsewhere} \end{aligned}$$

Y has a probability density function of $f_Y(y) = \frac{y^3}{64}$ for $0 \le y \le 4$.

Calculate $E[4X+5Y]$.

64. You roll a fair 6 sided die three times. Let X be the number of 6s that are rolled.

   a. Calculate the $E[X]$ using the definition of Expectation.

   b. Calculate the $E[X]$ using the Indicator random variables.

65. A cookie jar that has five cookies of which 3 are chocolate chip and 2 are oatmeal. Gayathri randomly selects a cookie from the cookie jar. After she selects a cookie, the cookie jar is replenished by adding one cookie which is the same type as the cookie that she selected. Gayathri selects two more cookies where the same process is followed. (This means that this a problem with replacement.)

Let X be the number of chocolate chip cookies that Gayathri selects.

   a. Calculate the $E[X]$.

b. Calculate the $E[X]$ if there is no replacement of the cookies.

66. You are given that the random variable X is has a probability mass function of:

$$\begin{array}{ll} 0.2 & \text{if } x=3 \\ 0.3 & \text{if } x=8 \\ 0.5 & \text{if } x=10 \\ 0 & \text{elsewhere} \end{array}$$

Calculate the $Var[X]$.

67. You are given that the random variable Y has a probability density function of

$f_Y(y) = \dfrac{y^3}{64}$ for $0 \le y \le 4$.

Calculate the $Var[Y]$.

68. You have a standard deck of playing cards. There are 52 cards. There are 4 suits. Each suit has cards numbered from 2 to 10 and then four face cards (jack, queen, king and ace). You randomly draw a card from the deck. I will pay you the number on the card. If you draw a face card, I will pay you 15 unless it is an ace in which case you will pay me 100.

Let X be the random variable representing the amount of the payment.

a. Calculate the $E[X]$ from your standpoint.

b. Calculate the $Var[X]$.

69. You are given that $E[X]=10$ and $Var[X]=40$. Calculate $E[(X+5)^2]$.

70. You are given that $E[X]=2,\ E[X^3]=9,\ and\ E[(X-2)^3]=0$.

Calculate $Var[X]$ and the standard deviation of X .

71. A recent study indicates that the annual cost of maintaining a car in West Lafayette averages 200 with a variance of 200. West Lafayette has just levied a 20% tax on car maintenance so the cost of everything if 120% of the prior cost.

Determine the expected value and variance of car maintenance in West Lafayette with the new tax in place.

72. A random variable X has the cumulative distribution function:

$$F_X(x) = \begin{cases} 0, & x < 2 \\ \dfrac{x^2 - 2x + 2}{10}, & 2 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

Calculate $Var[X]$.

73. The moment generating function for X is $M(t) = c + 0.4e^t + 0.3e^{2t} + 0.2e^{at}$ where c is a constant. The mean of the random variable is 2.

Determine a .

74. Let the random variable X have a moment generating function of $M(t) = e^{3t+t^2}$.

Calculate $E[X^2]$.

75. Let the random variable X have a moment generating function of $M(t) = \left(\dfrac{2+e^t}{3}\right)^9$.

Calculate $Var[X]$.

76. The damage to a house as a result of a tornado has a random variable X with a moment generating function of:

$$M(t) = \frac{1}{(1-2500t)^4}$$

Determine the standard deviation of the amount of damage.

77. You are given that the Probability Generating Function for the random variable X is

$$P_X(t) = \frac{t^1}{6} + \frac{t^2}{3} + \frac{t^3}{3} + \frac{t^4}{6}$$

a. Calculate $p(x)$ for $x = 1, 2, 3, 4$

b. Calculate $E[X]$.

c. Calculate $Var[X]$.

78. The random variable X is distributed as a discrete uniform distribution and can take on values of 1, 2, 3, …, 100.

    a. Calculate $E[X]$.

    b. Calculate $Var[X]$

79. The random variable X is distributed as a discrete uniform distribution and can take on values of 0, 1, 2, 3, …, 100.

    a. Calculate $E[X]$.

    b. Calculate $Var[X]$

80. The random variable X is distributed as a discrete uniform distribution and can take on values of 4, 8, 12, …, 300.

    a. Calculate $E[X]$.

    b. Calculate $Var[X]$

81. The Purdue freshman class has 10,000 students. The probability that a student will not return to campus next year is 0.15. A student's return to school is independent of any other student's return to school.

    Let R be the random variable for the number of students that will return to school.

    Calculate $E[R]$ and $Var[R]$.

82. You are given that X is distributed as a binomial distribution with parameters of $n=5$ and $p=0.3$.

    a. Calculate the probability that $X=2 \text{ or } 4$.

    b. Calculate $F_X(2)$.

83. The ice cream machine in Hillenbrand Hall has a probability of breaking down in any day of 0.20. The breakdowns are independent of any other breakdowns. The machine can only breakdown once per day.

    Calculate the probability that the machine breaks down two or more times in a 10 day period.

84. Two studies are being conducted on Covid vaccines. There are ten participants in each study. There is a 20% chance that a participant will not complete the study. Drop outs are independent of other drop outs.

    What is the probability that at least 9 participants will complete the study for one of the two groups but both groups will not have at least 9 participants complete the study.

85. You have three random variables which are distributed as follows:

   i. $X \sim Binomial(4, 0.4)$

   ii. $Y \sim Binomial(8, 0.4)$

   iii. $Z \sim Binomial(12, 0.4)$

   The random variable $S = X + Y + Z$.

   a. Calculate the $E[S]$.

   b. Calculate the $Var[S]$.

86. A bowl has four tiles numbered 1, 2, 3, and 4. You randomly draw a tile from the bowl and note the number. You replace the tile and draw again.

   A success is if the sum of the two tiles exceeds 6.

   N is the number of times you repeat the experiment when you have your first success.

   a. Calculate the probability that $N = 4$.

   b. Calculate $E[N]$.

   c. Calculate $Var[N]$.

87. A part of a wellness check for faculty at Purdue, all faculty members are being tested for high blood pressure. Let X be the number of tests completed when the first person with high blood pressure is found. The $E[X] = 12.5$.

   Calculate the probability that the sixth person tested is the first person with high blood pressure.

88. A fair coin is tossed repeatedly. Calculate the probability that the third head occurs on the nth toss.

89. A bowl has five tiles numbered 1, 2, 3, 4 and 5. You randomly draw a tile from the bowl and note the number. You replace the tile and draw again.

   A success is if the sum of the two tiles exceeds 7.

   X is the number of times you repeat the experiment when you have your third success.

   a. Calculate $E[X]$.

   b. Calculate $Var[X]$.

90. In a shipment of 20 packages, 7 packages are damaged. The packages are randomly inspected, one at a time, without replacement, until the fourth damaged package is discovered.

    Calculate the probability that exactly 12 packages are inspected.

91. Let X be the number of independent Bernoulli trials performed until a success occurs. Let Y be the number of independent Bernoulli trials performed until 5 successes occur. A success occurs with a probability of p and the $Var[X]=3/4$.

    Calculate the $Var[Y]$.

92. You are given that X is distributed as a Poisson distribution with $\lambda=1$.

    a. Calculate the probability that $X \geq 2$.

    b. Calculate that probability that $X \geq 2$ given that $X \leq 4$.

93. The number of traffic accidents per week in Remington has a Poisson distribution with a mean of 3.

    What is the probability of exactly two accidents in two weeks.

94. The number of power surges in an electric grid has a Poisson distribution with a mean of 1 surge every 12 hours.

    Calculate the probability that there will be no more than one power surge in a 24 hour period.

95. You are given that N is distributed as Poisson. You are also given that $\Pr(N=2)=3\Pr(N=4)$.

    Calculate the $Var[N]$.

96. The number of people who ride an elevator in the Math Building is distributed as a Poisson distribution with an average of 60 people per hour. Students make up 80% of those riding the elevator while faculty make up the other 20%

    Calculate the probability that no faculty member rides the elevator during a 15 minute period.

97. Let the random variable L be uniformly distributed between 2000 and 20,000.

    a. Calculate the $E[L]$.

    b. Calculate the $Var[L]$.

    c. Calculate the probability that L will exceed 15,000.

    d. Calculate the probability that L will exceed 15,000 given that it exceeds 12,000.

98. The expected inter-arrival time of busses at the Greyhound station in Indianapolis is 20 minutes.

   a. Determine the parameter θ for the exponential distribution.

   b. Calculate the probability that the time between busses will be at least 20 minutes.

   c. Calculate the probability that the time between busses will exceed 20 minutes but will be less than 30 minutes.

   d. Calculate the variance of the inter-arrival time.

   e. If you arrive at the bus station 10 minutes after the last bus arrived, what is the expected time until the next bus arrives.

99. The lifetime of a printer is exponentially distributed with a mean of 2 years. The printer costs 200. The manufacturer of the printer provides a guarantee that it will refund the cost of the printer if it fails in the first year. Additionally, the manufacturer will refund 50% of the cost if the printer fails in the second year. No refund is provided if the printer does not fail in the first two years.

    Calculate the expected total amount of refunds for each printer sold.

100. The time to failure of an electronic component has an exponential distribution with a median of four hours. Remember that the median is the x such that $F_X(x) = 0.5$.

    Calculate the probability that the component will work without failing for at least 5 hours.

101. A company has two independent electric generators. The time until failure for each generator follows an exponential distribution with a mean of 10. The company will begin using the second generator upon the failure of the first generator.

    Calculate the variance of expected total time that the generators produce electricity.

102. Calculate $\Gamma(6)$.

103. The random variable X has a Gamma distribution with parameters $\alpha = 6$ and $\beta = 0.5$.

   a. Calculate $E[X]$.

   b. Calculate $Var[X]$.

104. The random variable Y has Gamma distribution with a mean of 24 and a variance of 12.

    Determine the parameters α and β .

105. Let X be distributed as a Gamma distribution with $\alpha = 6$ and $\beta = 3$ and Y be distributed as a Gamma distribution with $\alpha = 21$ and $\beta = 3$.

The random variable $S = X + Y$.

Calculate the $Var[S]$.

106. If $X \sim Normal(5,16)$, calculate the $\Pr(X > 7)$.

107. Let X be a normal random variable with a mean of zero and a variance of $a > 0$. Calculate the $\Pr(X^2 < a)$.

108. Let $X_1, X_2, ..., X_{36}$ be an random sample from a distribution with mean $\mu_X = 30.4$ and standard deviations of $\sigma_X = 12.$

Using the Central Limit Theorem, calculate the approximate value of $\Pr[\bar{X}_{36} > 29].$

109. Let $X_1, X_2, ..., X_{36}$ and $Y_1, Y_2, ..., Y_{49}$ be independent random samples from distributions with means $\mu_X = 30.4$ and $\mu_Y = 32.1$. Further, the distributions also have standard deviations of $\sigma_X = 12$ and $\sigma_Y = 14.$

Using the Central Limit Theorem, calculate the approximate value of $\Pr[\bar{X}_{36} > \bar{Y}_{49}].$

110. Let $X_1, X_2, ..., X_{100}$ be a random sample from an exponential distribution with a mean of 0.5.

Determine the approximate value of $\Pr\left[\sum_{i=1}^{100} X_i > 57\right]$ using the Central Limit Theorem.

111. Let $X_1, X_2, ..., X_n$ be an random sample from a distribution with mean $\mu_X = 30.4$ and standard deviations of $\sigma_X = 12.$

Using the Central Limit Theorem, determine the number of samples necessary to be 95% certain that $\bar{X}_n$ is within 1 of the true value of μ_X .

112. Let the random variable N be distributed as Poisson with an expected value of 81.

Use the normal distribution to approximate the probability that N will exceed 100.

113. Let the random variable N be distributed as Poisson with an expected value of 81.

Use the normal distribution to approximate A such that $\Pr[81 - A < N < 81 + A] = 0.80.$

114. An insurance company sells 500 auto insurance policies. The number of claims per year for each policy follows a Poisson distribution with a mean of 0.2 claims. The number of claims between policies is independent.

Using the normal distribution to approximate the probability that the company will have 80 claims or more.

115. An insurance company issues 1250 vision care policies. The number of claims filed by a policyholder under a vision care policy during one year is Poisson with a mean of 2. The number of claims filed by each policy is independent of other policyholders.

Using the normal distribution of approximate that the total number of claims will be between 2450 and 2600 during a one year period.

116. You are given that the Random Variable X can have values of 1, 2, 3 and random variable Y can have values of 1, 2, 3, 4. Further, the table below lists the joint probability mass function.

	X		
Y	1	2	3
1	1/10	1/20	1/5
2	0	1/10	1/20
3	1/20	1/10	1/10
4	3/20	0	1/10

a. Calculate $\Pr[X = 2, Y = 3]$.
b. Calculate the marginal pdf of X .
c. Calculate the marginal pdf of Y .
d. Calculate the $\Pr[X + Y > 4]$.
e. Calculate $F_{X,Y}(1,2)$.

117. Let X and Y be discrete random variables with joint density function:

$$f_{X,Y}(x,y) = \frac{1}{6} \quad \text{for } x = 1,2,3 \text{ and } y = 2,3. \quad f_{X,Y}(x,y) = 0 \text{ elsewhere.}$$

Let $U = X + Y$. Calculate the probability mass function for U .

118. Let X and Y be discrete random variables with Joint probability function:

$$f_{X,Y}(x,y)=\begin{cases}\dfrac{2x+y}{12}, & \text{for } (x,y)=(0,1),(0,2),(1,2),(1,3)\\ 0, & \text{elsewhere}\end{cases}$$

Determine the marginal probability function for X.

119. The continuous random variable X and Y have a joint probability density function of $k(x+y)$ for $0<x<2$ and $0<y<1$.

a. Determine k .

b. Calculate $F_{X,Y}(0.4,0.5)$.

c. Calculate $\Pr[X+Y>2]$.

d. Calculate $\Pr[X+Y>0.7]$.

e. Determine the margin pdf of x .

120. Let X and Y continuous random variables with joint density function:

$$f_{X,Y}(x,y)=\begin{cases}\dfrac{12}{25}\left(x+y^2\right), & \text{for } 1<x<y<2\\ 0, & \text{elsewhere}\end{cases}$$

Determine the marginal density function of Y.

121. Let X and Y be continuous random variables with a joint density function of $f_{X,Y}(x,y)=e^{-(x+y)},\ x>0,\ y>0$.

Calculate the $\Pr[X+Y<1]$.

122. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f_{X,Y}(x,y)=\frac{x+y}{8}, \quad \text{for } \ 0<x<2,\ 0<y<2 \ .$$

Calculate the probability that the device fails in the first hour of operation.

123. Let X and Y be continuous random variables with joint density function:

$$f_{X,Y}(x,y)=\begin{cases} xy, & \text{for } 0\le x\le 2 \ \text{ and } \ 0\le y\le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate the $\Pr[X/2\le Y\le X]$.

124. Let X and Y continuous random variables be independent with the following density functions:

$$f_X(x)=1, \quad \text{for } 0<x<1 \quad \text{and} \quad f_Y(y)=2y, \quad \text{for } 0<y<1 \ .$$

Calculate the $\Pr[X<Y]$.

125. Let X and Y continuous random variables with joint density function:

$$f_{X,Y}(x,y)=\begin{cases} \frac{12}{25}\left(x+y^2\right), & \text{for } 1<x<y<2 \\ 0, & \text{elsewhere} \end{cases}$$

Calculate $E[X+Y]$.

126. Let X and Y be continuous random variables with a joint density function of:

$$f_{X,Y}(x,y)=0.25, \quad \text{for } \ 0\le x\le 2,\ \ x-2\le y\le x \qquad \text{and 0 elsewhere}$$

Calculate $E[X^3Y]$.

127. You are given that the Random Variable X can have values of 1, 2, 3 and random variable Y can have values of 1, 2, 3, 4. Further, the table below lists the joint probability mass function.

	X		
Y	1	2	3
1	1/10	1/20	1/5
2	0	1/10	1/20
3	1/20	1/10	1/10
4	3/20	0	1/10

a. Calculate $E[X+Y]$.

b. Calculate $E[Min(X,Y)]$.

128. The profit on a new product is given by $Z=3X-Y-5$. Furthermore, you are given that X and Y are independent with $Var[X]=1$ and $Var[Y]=2$.

Calculate the $Var[Z]$.

129. The profit on a new product is given by $Z=5X-2Y-3$. Furthermore, you are given that $Var[X]=1$, $Var[Y]=2$, and $Cov[X,Y]=-1$.

a. Calculate the $Var[Z]$.

b. Calculate $\rho(X,Y)$.

130. The continuous random variables X and Y have a joint density function of

$$f_{X,Y}(x,y)=6x, \quad \text{for } 0<x<y<1 \quad \text{and} \quad 0 \text{ elsewhere.}$$

You are also given that $E[X]=1/2$ and $E[Y]=3/4$.

Calculate $Cov[X,Y]$.

131. The continuous random variables X and Y have a joint density function of

$$f_{X,Y}(x,y)=\frac{8xy}{3}, \quad \text{for } 0\le x\le 1 \text{ and } x<y<2x \quad \text{and} \quad 0 \text{ elsewhere.}$$

Calculate $Cov[X,Y]$.

132.

Answers

1.
    a. $\{1,2,3,5\}$
    b. $\{1,3\}$
    c. $\varnothing$
    d. $\{4,5,6\}$
    e. $\{1\}$
    f. $\{2,3\}$
    g. Not Given
2.
    a. 0.6
    b. 0.7
    c. 0.4
    d. 0.1
    e. 0.8
3. 5/6
4. 0.6
5. 0.5
6. 0.05
7. 0.15
8.
    i. 26
    ii. 8
    iii. 28

9.
   a. 5/36
   b. 1/18
   c. 5/18
   d. 1/6
10.
   a. 2,600,000
   b. 786,240
11. 120
12. 60
13. 27/60
14.
   a. 53
   b. 0.15
15. 7560
16. 14,400
17. 5/12
18. 0.000305
19. 7/10
20. 0.18
21. 1/24
22. 25/87
23. 22/120
24. 15/154
25. No Answer Given
26.
   a. 5/108
   b. 2/3
27. 1/7

28. 0.4
29. 0.3
30. 17/38
31. 2/3
32. 0.34
33. 7/30
34. 19/216
35. ¾
36. 23/24
37. 108/625
38. 0.32842
39. 7/15
40.
    a. No Answer Given
    b. No Answer Given
    c. 2/9
    d. ¼
    e. No Answer Given
41. 7000
42. 1/6
43. 0.657439
44. 0.824/1.704
45. 1/6
46. 19/63
47. 0.4276
48. 0.46875
49. $\frac{12.5}{x+10} - 0.25$
50. 0.15

51. 6.5
52. 16,376
53. 29,898
54. 6
55. 4/3
56. $\dfrac{17}{24\sqrt{\theta}}$
57. 28/15
58. 3
59. 0.52822
60. 0.15642
61. 0.991597
62. 7/15
63. 48
64.
    a. ½
    b. ½
65.
    a. 9/5
    b. 9/5
66. 7
67. 32/75
68.
    a. -1/13
    b. 850.686
69. 265
70. 1/6 and $\sqrt{1/6}$
71. 240 and 288
72. 104/225

73. 5
74. 11
75. 2
76. 5000
77.
    a. Not given
    b. 2.5
    c. 11/12
78.
    a. 50.5
    b. 833.25
79.
    a. 50
    b. 850
80.
    a. 152
    b. 7498.67
81. 8500 and 1275
82.
    a. 0.33705
    b. 0.83692
83. 0.6242
84. 0.469
85.
    a. 9.6
    b. 5.76

86.

a. 0.10057
b. 16/3
c. 208/9

87. 0.052727

88. $\binom{n-1}{2}\left(\frac{1}{2}\right)^n$

89.

a. 12.5
b. 475/12

90. 0.119195

91. 3.75

92.

a. 0.26424
b. 17/65

93. 0.044618

94. 0.40601

95. 2

96. 0.049787

97.

a. 11,000
b. 27,000,000
c. 5/18
d. 5/8

98.

a. 1/20
b. 0.368
c. 0.14475
d. 400

e. 20

99. 102.56

100. 0.42045

101. 200

102. 120

103.

a. 12

b. 24

104. $\alpha = 48$ and $\beta = 2$

105. 3

106. 0.3085

107. 0.6826

108. 0.7580

109. 0.274

110. 0.0808

111. 554

112. 0.0174

113. 11.54

114. 0.9772 or 0.9798 With the Continuity Correction

115. 0.8185 or 0.8216

116.

a. 1/10

b. p(1)=6/20; p(2)=5/20; p(3)=9/20

c. p(1)=7/20; p(2)=3/20; p(3)=5/20; p(4)=5/20

d. 10/20 = 0.5

e. 1/10

117. $$p(U)=\begin{cases}\frac{1}{6}, \text{ if } u=3\\ \frac{1}{3}, \text{ if } u=4\\ \frac{1}{3}, \text{ if } u=5\\ \frac{1}{6}, \text{ if } u=6\end{cases}$$

118. 0.25 for x=0 and 0.75 for x=1

119.

a. 1/3

b. 0.03

c. 7/18

d. 0.96189

e. $\frac{1}{3}\left(x+\frac{1}{2}\right)$

120. $\frac{12}{25}y^3-\frac{6}{25}y^2-\frac{6}{25}$ for $1<y<2$ and 0 elsewhere

121. $1-2e^{-1}$

122. 5/8

123. 3/8

124. 2/3

125. 3.084

126. 6/5

127.

a. 4.55

b. 1.65

128. 11

129.

a. 53

b. $\frac{-1}{\sqrt{2}}$

130. 1/40

131. 0.0415