STAT 472
Fall 2018
Quiz 2

September 5, 2018

1. You are given that $\mu_{50+t} = 0.01 + 0.004t$.

   Calculate ${}_{3|4}q_{50}$.

   Solution:

$$ {}_{t}p_{50} = e^{-\int_0^t \mu_{50+s}ds} = e^{-\int_0^t (0.01+0.004s)ds} = e^{-0.01t-0.002t^2} $$

$$ {}_{3|4}q_{50} = {}_{3}p_{50} - {}_{7}p_{50} = e^{-0.01(3)-0.002(9)} - e^{-0.01(7)-0.002(49)} = 0.10778 $$

2. You are given that ${}_t p_{95} = 1 - 0.04t^2$ for $0 \le t \le 5$

Calculate $Var[K_{95}]$.

Solution:

$$Var[K_{95}] = E[{K_{95}}^2] - (E[K_{95}])^2$$

$$E[K_{95}] = \sum_{k=1}^{4} {}_k p_{95} = \sum_{k=1}^{4} (1 - 0.04k^2)$$

$$= 1 - 0.04(1^2) + 1 - 0.04(2^2) + 1 - 0.04(3^2) + 1 - 0.04(4^2) = 2.8$$

$$E[{K_{95}}^2] = 2\sum_{k=1}^{4} k \cdot {}_k p_{95} - E[K_x] = 2\sum_{k=1}^{4} k(1 - 0.04k^2) - 2.8$$

$$= 2\left\{1\left[1 - 0.04(1^2)\right] + 2\left[1 - 0.04(2^2)\right] + 3\left[1 - 0.04(3^2)\right] + 4\left[1 - 0.04(4^2)\right]\right\} - 2.8 = 9.2$$

$$Var[K_{95}] = 9.2 - (2.8)^2 = 1.36$$

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1. You are given that $S_0(x) = \frac{8100 - x^2}{8100}$ for $0 \le x \le 90$.

   Calculate ${}_{3|4}q_{50}$.

   Solution:

$$ {}_t p_{50} = \frac{S_0(50+t)}{S_0(50)} = \frac{8100-(50+t)^2}{8100-50^2} = \frac{5600-100t-t^2}{5600} $$

$$ {}_{3|4}q_{50} = {}_3 p_{50} - {}_7 p_{50} = \frac{5600-100(3)-3^2}{5600} - \frac{5600-100(7)-7^2}{5600} = 0.07857 $$

2. You are given that ${}_tq_{75} = \dfrac{t^2+t}{240}$ for $0 \le t \le 15$.

Calculate $Var[T_{75}]$

Solution:

$$Var[T_{75}] = E[T_{75}{}^2] - \left(E[T_{75}]\right)^2$$

$$E[T_{75}] = \int_0^{15} {}_tp_{75} \cdot dt = \int_0^{15} (1 - {}_tq_{75}) \cdot dt = \int_0^{15} \left[1 - \frac{t^2+t}{240}\right] \cdot dt$$

$$= \left[t - \frac{t^3}{720} - \frac{t^2}{480}\right]_0^{15} = 15 - \frac{15^3}{720} - \frac{15^2}{480} = 9.84375$$

$$E[T_{75}{}^2] = 2\int_0^{15} t \cdot {}_tp_{75} \cdot dt = 2\int_0^{15} t(1 - {}_tq_{75}) \cdot dt = 2\int_0^{15} \left[t\left(1 - \frac{t^2+t}{240}\right)\right] \cdot dt$$

$$= 2\left[\frac{t^2}{2} - \frac{t^4}{960} - \frac{t^3}{720}\right]_0^{15} = 2\left[\frac{15^2}{2} - \frac{15^4}{960} - \frac{15^3}{720}\right] = 110.15625$$

$$Var[T_{75}] = 110.15625 - (9.84375)^2 = 13.25684$$